Efficiency in the Bitcoin Market

Zichao Yang

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Motivation

The differences between bitcoin market and stock market:

- Bitcoin market is thinner than stock market.
- Bitcoin market does not have a real close or open time.
- There are hundreds of bitcoin exchanges operating in the world.
- Algorithm plays a role in deciding the pattern of bitcoin price.

In this paper, we focus on the last two points.
What does this paper do?

We take the technical aspect of bitcoin into consideration and study how the transaction fee functions differently in the bitcoin market.

We argue that there are two layers of efficiency in the bitcoin market:

- Efficiency on exchange level.
- Efficiency on the whole market level.

These two types of efficiency do not always move in the same direction.
(1) Studies about the predictability of bitcoin return:

- People’s attention and bitcoin return: results are inconclusive. (Kristoufek, 2015; Ciaian et al., 2016; Urquhart, 2018; Aalborg et al., 2018; Dastgir et al., 2019)

- Political or economic uncertainty and bitcoin return: results are inconclusive. (Dyhrberg, 2016; Ciaian et al., 2016; Bouri et al., 2017; Demir et al., 2018; Al-Khazali et al., 2018)

- Trade volume and bitcoin return: results are inconclusive. (Balcilar et al., 2017; Bouri et al., 2018; El Alaoui et al., 2018; Kjærland et al., 2018)
Literature review

(2) Studies testing weak form efficiency:

The results are inconclusive. Bitcoin is the most efficient one when compared with alter-coins, and efficiency is positively related with liquidity. (Urquhart, 2016; Nadarajah and Chu, 2017; Zhang et al., 2018; Bouri et al., 2018)

(3) Price across exchanges

- Brandvold et al. (2015) find that, in the period from April 2013 to February 2014, big bitcoin exchanges provide more information on price.

- Giudici and Abu-Hashish (2018) find that bitcoin price is highly interrelated among different exchanges.
Blocks and the blockchain

**Blocks** are defined as storage units which contain confirmed transaction data.

**Blockchain** is a sequence of blocks, where each block contains a piece of information about the location of its previous block.

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**Transaction** View information about a bitcoin transaction

<table>
<thead>
<tr>
<th>Summary</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>246 (bytes)</td>
</tr>
<tr>
<td>Weight</td>
<td>654</td>
</tr>
<tr>
<td>Received Time</td>
<td>2019-04-03 19:38:21</td>
</tr>
<tr>
<td>Lock Time</td>
<td>Block: 570071</td>
</tr>
<tr>
<td>Included In Blocks</td>
<td>570072 (2019-04-03 19:49:25 + 11 minutes)</td>
</tr>
<tr>
<td>Confirmations</td>
<td>1</td>
</tr>
<tr>
<td>Visualize</td>
<td>View Tree Chart</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inputs and Outputs</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Input</td>
<td>0.26226692 BTC</td>
</tr>
<tr>
<td>Total Output</td>
<td>0.19830692 BTC</td>
</tr>
<tr>
<td>Fees</td>
<td>0.06396 BTC</td>
</tr>
<tr>
<td>Fee per byte</td>
<td>26,000 sat/B</td>
</tr>
<tr>
<td>Fee per weight unit</td>
<td>9,779.817 sat/WU</td>
</tr>
<tr>
<td>Estimated BTC Transacted</td>
<td>0.00410706 BTC</td>
</tr>
<tr>
<td>Scripts</td>
<td>Hide scripts &amp; coinbase</td>
</tr>
</tbody>
</table>
Mining and Proof-of-Work

The structure of block header:

<table>
<thead>
<tr>
<th>Field</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Version</td>
<td>A version number to track software/protocol upgrades</td>
</tr>
<tr>
<td>Previous Block Hash</td>
<td>A reference to the hash of the previous block in the chain</td>
</tr>
<tr>
<td>Merkle Root</td>
<td>A hash of the root of the merkle tree of this block’s transactions</td>
</tr>
<tr>
<td>Timestamp</td>
<td>The approximate creation time of this block</td>
</tr>
<tr>
<td>Target</td>
<td>The Proof-of-Work algorithm target for this block</td>
</tr>
<tr>
<td>Nonce</td>
<td>A counter used for the Proof-of-Work algorithm</td>
</tr>
</tbody>
</table>

This table is adopted from Table 10-3 in Antonopoulos (2017).

Block header hash: 0000000000000000ae165831e934ff763ae46a2a6c172b3f1b60a8ce26f

Mining: a process of hashing the block header repeatedly. In each time, the miner change the value of nonce a little bit until the block header hash satisfies the target set by the **Proof-of-Work** algorithm.
More on Proof-of-Work

- Difficult to find the nonce, but easy to verify the finding
- Makes the blockchain almost immutable (with mining competition)
- Auto-adjustable, generates blocks, on average, every 10 minutes
- Adjustment not symmetric [See graph]
Exchange trade volume

<table>
<thead>
<tr>
<th>Time</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 BTC</td>
<td>y USD</td>
</tr>
<tr>
<td>2</td>
<td>x USD</td>
<td>1 BTC</td>
</tr>
</tbody>
</table>

Transaction fee I: charged by exchange
Transaction fee II: Charged by miner

Deposits US dollars

Transaction fee II

This transaction is recorded in blockchain

B’s private digital wallet \( \beta \)

Transaction fee II

This transaction is recorded in blockchain
On-chain transaction volume

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private key 1 (1 bitcoin)</td>
<td>Public key b (2.5 bitcoin)</td>
</tr>
<tr>
<td>Private key 2 (2 bitcoin)</td>
<td>Public key a1 (0.49 bitcoin)</td>
</tr>
</tbody>
</table>

*The difference between input and output (0.01 bitcoin) serves as transaction fee, which will be collected by miner.*

- **Wallet A**
  - Public key a1/private key 1
  - Public key a2/private key 2

- **Wallet B**
  - Public key b/private key 3
On-chain transaction volume

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private key 1</td>
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</tr>
<tr>
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</tr>
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<td>Public key x</td>
</tr>
<tr>
<td>(2 bitcoin)</td>
<td>(0.49 bitcoin)</td>
</tr>
</tbody>
</table>

The difference between Input and Output (0.01 bitcoin) serves as transaction fee, which will be collected by miner.

Wallet A
Public key a1/private key 1
Public key a2/private key 2

Wallet B
Public key b/private key 3
The procedure of transaction confirmation

Mempool
... Transaction a ...

Mempool
... Transaction a ...

Mempool
... Transaction a ...

Block k-2

Block k-1

Block k

Transaction 1
Transaction 2
.......
Transaction a
.......
Transaction n
Transaction fee

Transaction fees are left by users, then collected by miners.

When miners build their blocks, they select the transactions which have the highest reward-size ratio (RSR):

\[
RSR = \frac{\text{transaction fee paid in this transaction}}{\text{the size of this transaction data}}
\]

Users make transaction decisions based on the ratio of transaction value to transaction fee (TFR):

\[
TFR = \frac{\text{transaction fee paid in this transaction}}{\text{the value of this transaction}}
\]

The gap between the goals of miners and users is bridged by the bitcoin wallet.
Mempool size and transaction fee per transaction
Number of transactions and transaction fee per transaction
Tx fee per tx and total tx volume
Exchange-level data

Exchange-level data comes from CryptoCompare. It includes the bitcoin close price in the seven largest exchanges (Bitfinex, Bitstamp, Coinbase, Gemini, Kraken, Lakebtc and Quoine) and market trade volume data from 2015-10-08 to 2019-03-03.

<table>
<thead>
<tr>
<th></th>
<th>Bitfinex</th>
<th>Bitstamp</th>
<th>Coinbase</th>
<th>Gemini</th>
<th>Kraken</th>
<th>Lakebtc</th>
<th>Quoine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit: USD (dollar)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>3760.6</td>
<td>3749.2</td>
<td>3758.9</td>
<td>3755.4</td>
<td>3749.0</td>
<td>3973.3</td>
<td>3799.3</td>
</tr>
<tr>
<td>Median</td>
<td>2461.0</td>
<td>2502.0</td>
<td>2505.6</td>
<td>2497.6</td>
<td>2497.8</td>
<td>2598.3</td>
<td>2681.7</td>
</tr>
<tr>
<td>Maximum</td>
<td>19210.0</td>
<td>19187.8</td>
<td>19650.0</td>
<td>19500.0</td>
<td>19356.9</td>
<td>19315.7</td>
<td>19653.5</td>
</tr>
<tr>
<td>Minimum</td>
<td>243.6</td>
<td>243.4</td>
<td>243.4</td>
<td>243.9</td>
<td>242.7</td>
<td>240.6</td>
<td>244.5</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>3781.3</td>
<td>3786.5</td>
<td>3808.0</td>
<td>3800.49</td>
<td>3783.6</td>
<td>3991.3</td>
<td>3871.8</td>
</tr>
<tr>
<td>Observations</td>
<td>1243</td>
<td>1243</td>
<td>1243</td>
<td>1243</td>
<td>1243</td>
<td>1243</td>
<td>1243</td>
</tr>
</tbody>
</table>
Price efficiency in a single exchange

We apply the autoregressive analysis to study the dynamics of bitcoin return. The order of lag terms is chosen based on AIC. The autoregressive model can be written as:

\[ R_{i,t} = Const + A(L)R_{i,t} + \mu_t \]

where \( R_{i,t} \) denotes bitcoin return from exchange \( i \) at time \( t \). \( A(L) \) is lag operator and the order of lag terms is chosen based on AIC.

We combine the AR model with the rolling windows analysis (300-days window). Then we use an F-test to test the joint significance of all these lag terms in each regression. We count, in each period, how many exchanges have joint significant lag terms. There are seven exchanges, so the result in each period can range from 0 to 7.
Plot results with the mean of exchange trade volume
Price correlation among different exchanges

We still use bitcoin return data from the seven largest exchanges and conduct the test for Granger causality on the bitcoin return data for every two exchanges.

\[ R_{i,t} = Const + A(L)R_{i,t} + B(L)R_{j,t} + \mu_t \]

where \( R_{i,t} \) and \( R_{j,t} \) denote bitcoin return from exchange \( i \) and \( j \) at time \( t \), \( A(L) \) and \( B(L) \) are lag operators, which are chosen based on AIC.

We combine the Granger causality test with the rolling windows analysis. We calculate the number of correlated pairs (NCP) of exchanges by counting the number of exchanges, from which the return data can Granger cause the return of other exchanges in each period. In our dataset, there are seven different exchanges, so the maximum value of NCP is \( 7 \times 6 = 42 \).
Plot results with the inverse of CV of bitcoin price

The coefficient of variation (CV) of price can be defined as:

$$CV_t = \frac{\text{Standard Deviation of the price in } t^{th} \text{ period}}{\text{Mean of the price in } t^{th} \text{ period}}$$
Price dispersion and trade volume
Price dispersion and trade volume dispersion
Price dispersion and on-chain transaction volume

When on-chain trade volume is high, price dispersion becomes high and exchanges become more isolated with each other.
Daily data

Daily blockchain-related data come from CoinMetrics. The date range is chosen from 2015-01-01 to 2018-01-01.

Table 4: Summary statistics of blockchain-related data

<table>
<thead>
<tr>
<th></th>
<th>on-chain tx volume/tx</th>
<th>tx fee/tx</th>
<th>bitcoin return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>log-form</td>
<td>log-form</td>
<td>log-form</td>
</tr>
<tr>
<td>Mean</td>
<td>1.032</td>
<td>-7.998</td>
<td>0.003</td>
</tr>
<tr>
<td>Median</td>
<td>1.022</td>
<td>-8.211</td>
<td>0.003</td>
</tr>
<tr>
<td>Maximum</td>
<td>2.124</td>
<td>-5.539</td>
<td>0.224</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.250</td>
<td>-9.165</td>
<td>-0.236</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>0.310</td>
<td>0.752</td>
<td>0.038</td>
</tr>
<tr>
<td>Observations</td>
<td>1096</td>
<td>1096</td>
<td>1096</td>
</tr>
</tbody>
</table>
The structural VAR model is comprised of daily data on the log-form of bitcoin daily return \((R_t)\), the daily average on-chain transaction volume per transaction in BTC\((V_t)\), the daily average transaction fee per transaction in BTC\((F_t)\).

\[
\begin{bmatrix}
a_{11}^0 & 0 & 0 \\
a_{21}^0 & a_{22}^0 & 0 \\
a_{31}^0 & a_{32}^0 & a_{33}^0 \\
\end{bmatrix}
\begin{bmatrix} F_t \\ R_t \\ V_t \end{bmatrix} =
\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} +
\begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 \\
a_{21}^1 & a_{22}^1 & a_{23}^1 \\
a_{31}^1 & a_{32}^1 & a_{33}^1 \\
\end{bmatrix}
\begin{bmatrix} F_{t-1} \\ R_{t-1} \\ V_{t-1} \end{bmatrix} + \cdots +
\begin{bmatrix} \epsilon_F \\ \epsilon_R \\ \epsilon_V \end{bmatrix}
\]
Results
We match minute-level trade volume and price data from Bitstamp with block-level total transaction fees and number of transactions data from BlockSci.

Table 5: Summary statistics of blockchain-related data on block level

<table>
<thead>
<tr>
<th></th>
<th>Bitstamp tx volume/tx</th>
<th>tx fee/tx</th>
<th>bitcoin return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>log-form</td>
<td>log-form</td>
<td>log-form</td>
</tr>
<tr>
<td>Mean</td>
<td>-4.600</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Median</td>
<td>-4.352</td>
<td>-0.003</td>
<td>0.000</td>
</tr>
<tr>
<td>Maximum</td>
<td>5.992</td>
<td>6.329</td>
<td>0.070</td>
</tr>
<tr>
<td>Minimum</td>
<td>-18.787</td>
<td>-6.290</td>
<td>-0.152</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>2.037</td>
<td>0.411</td>
<td>0.004</td>
</tr>
<tr>
<td>Observations</td>
<td>150,785</td>
<td>150,785</td>
<td>150,785</td>
</tr>
</tbody>
</table>
Results
Mechanism behind results

- Increase in total tx vol
- Price dispersion increases
Mechanism behind results

Return increase

Increase in total tx vol

Price dispersion increases
Conclusion

Mechanism behind results

Limited capacity

Return increase

Profitable tx (miners)

Increase in total tx vol

Price dispersion increases

High tx fee

High value tx (users)

Increase in tx fee per tx

Increase in total tx vol

Costly arbitrage

Limited capacity

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Mechanism behind results

- Limited capacity
- Return increase
- Profitable tx (miners)
- Increase in tx fee per tx
- Increase in total tx vol
- Price dispersion increases
Mechanism behind results

- Limited capacity
  - Return increase
  - Profitable tx (miners)
    - Increase in tx fee per tx
- High tx fee
  - Increase in total tx vol
  - Price dispersion increases
Conclusion

Mechanism behind results

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Return increase

Profitable tx (miners)

Increase in tx fee per tx

TFR rule

High tx fee

High value tx (users)

Increase in total tx vol

Price dispersion increases

Efficiency in the Bitcoin Market

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May 28, 2019
Mechanism behind results

Limited capacity

- Return increase
- Profitable tx (miners)
  - TFR rule
  - High tx fee
  - High value tx (users)
    - Increase in tx fee per tx
    - Increase in tx vol per tx
    - Increase in total tx vol

Price dispersion increases
Mechanism behind results

Limited capacity

Return increase

Profitable tx (miners)

Increase in tx fee per tx

TFR rule

High tx fee

High value tx (users)

Increase in tx vol per tx

Increase in total tx vol

High tx fee

Price dispersion increases
Mechanism behind results

Limited capacity

Return increase → Profitable tx (miners) → Increase in tx fee per tx

TFR rule

High tx fee → High value tx (users) → Increase in tx vol per tx → Increase in total tx vol

High tx fee → Costly arbitrage → Price dispersion increases
Extensions

- Get more accurate block-level on-chain transaction volume data from BlockSci.
- Introduce structural breaks into our analysis.
- Use local projection method as a robustness check.
Future researches

- Cryptocurrency Confidence Survey
- A theoretical multi-exchanges model
- A network model using the data from BlockSci (block-level individual/clustered data)
BlockSci: how many clusters are involved in every block

BlockSci: the value of transactions in every block

Hash function

Plain Text: arbitrary length
Hashed Text: certain length (like 32 bytes)
Difficulty and computing power (hash rate)
On-chain transaction volume VS. Exchange trade volume

We match daily on-chain transaction volume data from CoinMetrics with minute-level trade volume data from Bitstamp. The correlation is 0.886.