Inspecting the mechanism

An analytical approach to the stochastic growth model

John Y. Campbell*

Princeton University, Princeton, NJ 08544, USA

This paper argues that a clear understanding of the stochastic growth model can best be achieved by working out an approximate analytical solution. The proposed solution method replaces the true budget constraints and Euler equations of economic agents with loglinear approximations. The model then becomes a system of loglinear expectational difference equations, which can be solved by the method of undetermined coefficients. The paper uses this technique to study shocks to techno-logy and shocks to government spending financed by lump-sum or distortionary taxation. It emphasizes that the persistence of shocks is an important determinant of their macroeconomic effects.

1. Introduction

During the last ten years, the stochastic growth model has become a work-horse for macroeconomic analysis. Perhaps the most forceful claims for the model have been made by Prescott (1986), who describes it as 'a paradigm for macro analysis – analogous to the supply and demand construct of price theory'. He also refers to the predictions of the model as those of 'standard economic theory'. In Prescott's view the shocks to the economy are random variations in the rate of technical progress, but the usefulness of the stochastic growth model does not depend on this view of the sources of business cycles. Other authors

have subjected the model to other types of shocks, for example government spending [Aiyagari, Christiano, and Eichenbaum (1992), Baxter and King (1993), Christiano and Eichenbaum (1992)], distortionary taxation [Baxter and King (1993), Braun (1993), Greenwood and Huffman (1991), McGrattan (1993)], and nominal shocks in the presence of sticky nominal wages and prices [King (1991)] or liquidity effects [Christiano and Eichenbaum (1991)]. The stochastic growth model enables one to track the dynamic effects of any shock; in this sense it is indeed a paradigm for macroeconomics.

Despite the wide popularity of the stochastic growth model, there is no generally agreed procedure for solving it. The difficulty is the fundamental nonlinearity that arises from the interaction between multiplicative elements, such as Cobb–Douglas production with labor and capital, and additive elements, such as capital accumulation and depreciation. This nonlinearity disappears only in the unrealistic special case where capital depreciates fully in a single period and agents have log utility [Long and Plosser (1983), McCallum (1989)]. In this case the model becomes loglinear and can be solved analytically. In all other cases, some approximate solution method is required.

In a seminal contribution, Kydland and Prescott (1982) proposed taking a linear-quadratic approximation to the true model around a steady-state growth path. Christiano (1988) and King, Plosser, and Rebelo (1987) have used a loglinear-quadratic approximation instead. This has at least two advantages: First, it delivers the correct solution in the special case that can be solved exactly, and second, it gives a simpler relation between the parameters of the underlying model and the parameters that appear in the approximate solution. Many other methods are also available, and have recently been reviewed and compared by Taylor and Uhlig (1990). Most of these methods are heavily numerical rather than analytical. While computational costs are no longer an important objection to numerical methods, the methods are often mysterious to the noninitiate and seem to bear little relation to familiar techniques for solving linear rational expectations models. A typical paper in the real business cycle literature states the model, then moves directly to a discussion of the properties of the solution without giving the reader the opportunity to understand the mechanism giving rise to these properties.[1]

In this paper I propose a simple analytical approach to the stochastic growth model. I start with the model's Euler equations and budget constraints; as Baxter (1991) has pointed out, this makes the approach applicable to models in which the competitive equilibrium is not Pareto optimal. Next I loglinearize the Euler equations and budget constraints in the manner of

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[1]The problem is also illustrated by Chapter 7 of Blanchard and Fischer (1989). Quite appropriately, this textbook confines itself to small macro models that can be solved analytically; lacking an appropriate solution method, Chapter 7 fails to convey the richness of the stochastic growth model or the real business cycle literature.

Campbell and Shiller (1988) and Campbell (1993). This transforms the model into a system of expectational difference equations in the capital stock and the exogenous variables driving the economy (here taken to be technology or government spending). I solve this system analytically using the method of undetermined coefficients.

There are important similarities, but also important differences, between this approach and the work of Christiano (1988) and King, Plosser, and Rebelo (1987). Christiano (1988) first substitutes all budget constraints into the objective function to set the model up as a calculus of variations problem. He then takes a second-order Taylor approximation in logs of the variables. Despite Christiano's use of a higher-order approximation, in a homoskedastic setting his method yields the same solution as the one obtained in this paper. The reason is that only expectations of second-order terms appear in Christiano's solution, and these expectations are constant if the model is homoskedastic. It follows that the evidence of Taylor and Uhlig (1990) and Christiano (1989) on numerical accuracy applies to the method of this paper as well. King, Plosser, and Rebelo (1987) write the model's first-order conditions using the Lagrange multiplier for the budget constraint as a state variable, and then loglinearize to obtain a system of expectational difference equations in the capital stock and the Lagrange multiplier. This is similar to the approach here, except that I use the capital stock and the exogenous driving variables as the state variables. This enables me to derive more directly the responses of endogenous variables to shocks in exogenous variables.

Perhaps the most important difference between this paper and previous work is that I solve the system of loglinear difference equations analytically in order to make the mechanics of the solution as transparent as possible. King, Plosser, and Rebelo (1987) instead solve the system using a general numerical method which can be more easily generalized to models with multiple state variables, but which obscures the simplicity of the basic stochastic growth model.

To illustrate the usefulness of the approach, this paper studies a number of issues in real business cycle analysis. Section 2 studies the effect of technology shocks in a model with fixed labor supply, showing how the insights of the literature on the permanent income hypothesis can be embedded in the stochastic growth model. Section 3 studies two alternative models of variable labor supply. In both sections the analytical solution method clarifies how the properties of the model depend on the parameters of the utility function and the persistence of technology shocks. As an illustration of the importance of persistence, the paper studies a slowdown in productivity growth of the type that seems to have occurred in the mid-1970's. Section 4 introduces shocks to government spending, again emphasizing the importance of persistence. This section also compares lump-sum taxation to gross output taxation as a means of government finance. Section 5 concludes.

2. A model with fixed labor supply

2.1. Specification of the model

The first equation of the model is a standard Cobb–Douglas production function. Using the notation Y_t for output, A_t for technology, and K_t for capital, and normalizing labor input $N_t = 1$, the production function is

$$Y_t = (A_t N_t)^\alpha K_t^{1-\alpha} = A_t^\alpha K_t^{1-\alpha}. \tag{1}$$

The second equation of the model describes the capital accumulation process:

$$K_{t+1} = (1 - \delta)K_t + Y_t - C_t, \tag{2}$$

where δ is the depreciation rate and C_t is consumption. Finally, there is a representative agent who maximizes the objective function

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}) = \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \frac{C_{t+i}^{1-\gamma}}{1 - \gamma}. \tag{3}$$

This time-separable power utility function with coefficient of relative risk aversion γ becomes the log utility function when $\gamma = 1$. I define the elasticity of intertemporal substitution $\sigma \equiv 1/\gamma$.

I also define a variable R_{t+1}, the gross rate of return on a one-period investment in capital, which equals the marginal product of capital in production plus undepreciated capital:

$$R_{t+1} \equiv (1 - \alpha)\left(\frac{A_{t+1}}{K_{t+1}}\right)^\alpha + (1 - \delta). \tag{4}$$

The first-order condition for optimal choice, given the objective function (3) and the constraints (1) and (2), can then be written in the simple form

$$C_t^{-\gamma} = \beta \mathrm{E}_t \{C_{t+1}^{-\gamma} R_{t+1}\}. \tag{5}$$

2.2. Steady-state growth

I now look for a steady-state or balanced growth path of this model, in which technology, capital, output, and consumption all grow at a constant common rate. I use the notation G for this growth rate: $G \equiv A_{t+1}/A_t$. In steady state the gross rate of return on capital R_{t+1} becomes a constant R, while the first-order

condition (5) becomes

$$G^\gamma = \beta R,\tag{6}$$

or in logs (denoted by lower-case letters),

$$g = \frac{\log(\beta) + r}{\gamma} = \sigma \log(\beta) + \sigma r.\tag{7}$$

This is the familiar condition relating the equilibrium growth rate of consumption to the intertemporal elasticity of substitution times the real interest rate in a model with power utility.

The definition of R (4) and the first-order condition (6) imply that the technology–capital ratio is a constant given by

$$\frac{A}{K} = \left[\frac{G^\gamma/\beta - (1 - \delta)}{1 - \alpha} \right]^{1/\alpha} \approx \left[\frac{r + \delta}{1 - \alpha} \right]^{1/\alpha}.\tag{8}$$

The first equality in (8) shows that a higher rate of technology growth leads to a lower capital stock for a given level of technology. The reason is that in steady state faster technology growth must be accompanied by faster consumption growth. Agents will accept a steeper consumption path only if the rate of return on capital is higher, which requires a lower capital stock. The second approximate equality in (8) comes from setting $G^\gamma/\beta = R \approx 1 + r$.

More generally, one can solve for various ratios of variables that will be constant along a steady-state growth path. I express these ratios in terms of four underlying parameters: g, the log technology growth rate; r, the log real return on capital; α, the exponent on labor and technology in the production function, or equivalently labor's share of output; and δ, the rate of capital depreciation. For purposes of 'calibration' in a quarterly model, benchmark values for these parameters might be $g = 0.005$ (2% at an annual rate), $r = 0.015$ (6% at an annual rate), $\alpha = 0.667$, and $\delta = 0.025$ (10% at an annual rate). Note that the rate of time preference β and the coefficient of risk aversion γ need not be specified, although (7) defines pairs of values for β and γ that are consistent with the assumed values of g and r.

Using the production function (1) and the formula for the technology–capital ratio (8), we have that the steady-state output–capital ratio is a constant,

$$\frac{Y}{K} = \left(\frac{A}{K} \right)^\alpha \approx \frac{r + \delta}{1 - \alpha}.\tag{9}$$

Similar reasoning shows that in steady state the consumption–output ratio is a constant,

$$\frac{C}{Y} = \frac{C/K}{Y/K} \approx 1 - \frac{(1-\alpha)(g+\delta)}{r+\delta}. \tag{10}$$

At the benchmark parameter values given above, the steady-state output–capital ratio $Y/K = 0.118$ (0.472 at an annual rate) and the steady-state consumption–output ratio $C/Y = 0.745$. These are fairly reasonable values.[2]

2.3. A loglinear model of fluctuations

Outside steady state, the real business cycle model is a system of nonlinear equations in the logs of technology, capital, output, and consumption. Nonlinearities are caused by incomplete capital depreciation [$\delta < 1$ in (2) and (4)] and by time variation in the consumption–output ratio. Thus *exact* analytical solution of the model is only possible in the unrealistic special case where capital depreciates fully in one period and where agents have log utility so the consumption–output ratio is constant [Long and Plosser (1983), McCallum (1989)]. The strategy of this section is instead to seek an *approximate* analytical solution by transforming the model into a system of approximate loglinear difference equations. For simplicity, all constant terms will be suppressed in the approximate model; the variables in the system can be thought of as zero-mean deviations from the steady-state growth path.

The Cobb–Douglas production function (1) needs no approximation; it can be written in loglinear form as

$$y_t = \alpha a_t + (1-\alpha)k_t, \tag{11}$$

where as always lower-case letters are used for log variables.

The capital accumulation equation (2) is unfortunately not loglinear. Dividing by K_t and taking logs, (2) becomes

$$\log[\exp(\Delta k_{t+1}) - (1-\delta)] = y_t - k_t + \log[1 - \exp(c_t - y_t)]. \tag{12}$$

The strategy proposed here is to take first-order Taylor approximations of the functions on the left- and right-hand sides of (12) around their steady-state values, and then to substitute out y_t using the log production function (11).

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[2]Simon (1990) briefly surveys alternative estimates of these ratios.

Calculations summarized in appendix A yield the following loglinear approximate accumulation equation:

$$k_{t+1} \approx \lambda_1 k_t + \lambda_2 a_t + (1 - \lambda_1 - \lambda_2)c_t, \tag{13}$$

where

$$\lambda_1 \equiv \frac{1 + r}{1 + g}, \qquad \lambda_2 \equiv \frac{\alpha(r + \delta)}{(1 - \alpha)(1 + g)}. \tag{14}$$

At the benchmark parameter values discussed above, $\lambda_1 = 1.01$, $\lambda_2 = 0.08$, and $1 - \lambda_1 - \lambda_2 = -0.09$. To understand these coefficients, one should note that $1 - \lambda_1 - \lambda_2 = -(C/Y)(Y/K)(1 + g)^{-1} = -(0.118)(0.745)(1.005)^{-1}$, the negative of the steady-state ratio of this period's consumption to next period's capital stock. A \$1 increase in consumption this period lowers next period's capital stock by \$1, but a 1% increase in consumption this period lowers next period's capital stock by only 0.09% because in steady state one period's consumption is only 0.09 times as big as the next period's capital stock.

I now turn to the general first-order condition (5). If the variables on the right-hand side of (5) are jointly lognormal and homoskedastic, then one can rewrite the first-order condition in log form as $E_t \Delta c_{t+1} = \sigma E_t r_{t+1}$, where $r_{t+1} \equiv \log(R_{t+1})$.[3] From the definition of the gross return on capital R_{t+1} in (4), the log return r_{t+1} is a nonlinear function of the log technology–capital ratio. The loglinear approximation of this function (calculated in appendix A) is

$$r_{t+1} \approx \lambda_3(a_{t+1} - k_{t+1}), \tag{15}$$

where

$$\lambda_3 \equiv \frac{\alpha(r + \delta)}{1 + r}. \tag{16}$$

At the benchmark parameter values discussed above, $\lambda_3 = 0.03$. This coefficient is extremely small. One way to understand this fact is to note that changes in technology have only small proportional effects on the one-period return on capital because capital depreciates only slowly, so most of the return is undepreciated capital rather than marginal output from the Cobb–Douglas

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[3]This uses the standard formula for the expectation of a lognormal random variable X_{t+1}: $\log(E_t X_{t+1}) \approx E_t \log(X_{t+1}) + \frac{1}{2}\text{var}_t \log(X_{t+1})$. Note that the assumption that the variables in the first-order condition are jointly lognormal and homoskedastic is consistent with a lognormal homoskedastic productivity shock and the approximations proposed here to solve the model.

production function. Alternatively, when δ is negligible (which it is not for the benchmark parameter values considered here), one could note that $r \approx R - 1 \approx (1 - \alpha)(A/K)^{\alpha}$. In this case a 1% increase in the technology–capital ratio raises r by about α%. But α% of r is only αr percentage points.

The representative agent's log first-order condition now becomes

$$E_t \Delta c_{t+1} = \sigma \lambda_3 E_t (a_{t+1} - k_{t+1}). \tag{17}$$

To close the model, it only remains to specify a process for the technology shock a_t. I assume that technology follows a first-order autoregressive or AR(1) process:

$$a_t = \phi a_{t-1} + \varepsilon_t, \qquad -1 \le \phi \le 1. \tag{18}$$

The AR(1) coefficient ϕ measures the persistence of technology shocks, with the extreme case of $\phi = 1$ being a random walk for technology.[4]

Eqs. (13), (17), and (18) form a system of loglinear expectational difference equations in technology, capital, and consumption. The parameters of these equations include λ_1, λ_2, and λ_3 (which are functions of the underlying growth parameters, r, g, α, and δ), the intertemporal elasticity of substitution σ, and the AR(1) coefficient ϕ that measures the persistence of technology shocks. The 'calibration' approach to real business cycle analysis takes λ_1, λ_2, and λ_3 as known, and searches for values of σ and ϕ (and a variance for the technology innovation) to match the moments of observed macroeconomic time series.

2.4. The method of undetermined coefficients

Eqs. (13), (17), and (18) can be solved using any of a number of standard methods. Here I use the method of undetermined coefficients. I adopt the notation η_{yx} for the partial elasticity of y with respect to x, and guess that log consumption takes the form

$$c_t = \eta_{ck} k_t + \eta_{ca} a_t, \tag{19}$$

where η_{ck} and η_{ca} are unknown but assumed to be constant. I verify this guess by finding values of η_{ck} and η_{ca} that satisfy the restrictions of the approximate loglinear model.

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[4]Eq. (18) suppresses a deterministic technology trend growing at rate g, since all variables in this section are measured as deviations from the steady-state growth path.

The conjectured solution can be written in terms of the capital stock, using (13), as

$$k_{t+1} = \eta_{kk}k_t + \eta_{ka}a_t,$$ (20)

where

$$\eta_{kk} \equiv \lambda_1 + (1 - \lambda_1 - \lambda_2)\eta_{ck}, \qquad \eta_{ka} \equiv \lambda_2 + (1 - \lambda_1 - \lambda_2)\eta_{ca}.$$ (21)

Also, substituting the conjectured solution into (17), I obtain

$$\eta_{ck}\Delta k_{t+1} + \eta_{ca}E_t\Delta a_{t+1} = \sigma\lambda_3 E_t a_{t+1} - \sigma\lambda_3 k_{t+1}.$$ (22)

Next I substitute (20) and (21) into (22) and use the fact that $E_t a_{t+1} = \phi a_t$. The result is an equation in only two state variables, the capital stock and the level of technology:

$$\eta_{ck}[\lambda_1 - 1 + (1 - \lambda_1 - \lambda_2)\eta_{ck}]k_t$$

$$+ \eta_{ck}[\lambda_2 + (1 - \lambda_1 - \lambda_2)\eta_{ca}]a_t + \eta_{ca}(\phi - 1)a_t$$

$$= \sigma\lambda_3\phi a_t - \sigma\lambda_3[\lambda_1 + (1 - \lambda_1 - \lambda_2)\eta_{ck}]k_t$$

$$- \sigma\lambda_3[\lambda_2 + (1 - \lambda_1 - \lambda_2)\eta_{ca}]a_t.$$ (23)

To solve this equation I first equate coefficients on k_t to find η_{ck}, and then equate coefficients on a_t to find η_{ca}, given η_{ck}.

Equating coefficients on k_t gives the quadratic equation

$$Q_2\eta_{ck}^2 + Q_1\eta_{ck} + Q_0 = 0,$$ (24)

where

$$Q_2 \equiv 1 - \lambda_1 - \lambda_2,$$

$$Q_1 \equiv \lambda_1 - 1 + \sigma\lambda_3(1 - \lambda_1 - \lambda_2),$$ (25)

$$Q_0 \equiv \sigma\lambda_3\lambda_1.$$

The quadratic formula gives two solutions to (24). With the benchmark set of parameters, one of these is positive. Eq. (13), with $\lambda_1 > 1$, shows that η_{ck} must be positive if the steady state is to be locally stable. Hence the positive solution is

the appropriate one:

$$\eta_{ck} = \frac{1}{2Q_2} \left\{ -Q_1 - \sqrt{Q_1^2 - 4Q_0Q_2} \right\}. \tag{26}$$

Note that η_{ck} depends only on σ and the λ parameters, and is invariant to the persistence of the technology shock as measured by ϕ. Solution of the model is completed by finding η_{ca} as

$$\eta_{ca} = \frac{-\eta_{ck}\lambda_2 + \sigma\lambda_3(\phi - \lambda_2)}{\phi - 1 + (1 - \lambda_1 - \lambda_2)(\eta_{ck} + \sigma\lambda_3)}. \tag{27}$$

2.5. Time-series implications

The consumption elasticities η_{ck} and η_{ca}, and the capital elasticities η_{kk} and η_{ka} derived from them, determine the dynamic behavior of the economy. Using lag operator notation, eq. (20) gives the capital stock as

$$k_{t+1} = \frac{\eta_{ka}}{(1 - \eta_{kk}L)} a_t. \tag{28}$$

Rewriting eq. (18) in the same notation, the technology process is

$$a_t = \frac{1}{(1 - \phi L)} \varepsilon_t. \tag{29}$$

These two equations imply that the capital stock follows an AR(2) process:

$$k_{t+1} = \frac{\eta_{ka}}{(1 - \eta_{kk}L)(1 - \phi L)} \varepsilon_t. \tag{30}$$

Two points are worth noting about this expression. First, the roots of the capital stock process are η_{kk} and ϕ, which are both real numbers. Thus, unlike the multiplier–accelerator model [Samuelson (1939)], the real business cycle model does not produce oscillating impulse responses. Second, the shock to capital at time $t + 1$ is the technology shock realized at time t. The capital stock is known one period in advance because it is determined by lagged investment and by a nonstochastic depreciation rate.

The stochastic processes for output and consumption are somewhat more complicated than the process for capital. The log production function (11) determines output as $y_t = (1 - \alpha)k_t + \alpha a_t$. In the fixed-labor model the partial

elasticities of output with respect to capital and technology are trivially $(1 - \alpha)$ and α. Substituting (29) and (30) into this expression, I obtain

$$y_t = \frac{(1 - \alpha)\eta_{ka}L}{(1 - \eta_{kk}L)(1 - \phi L)}\varepsilon_t + \frac{\alpha}{(1 - \phi L)}\varepsilon_t$$

$$= \frac{\alpha + [(1 - \alpha)\eta_{ka} - \alpha\eta_{kk}]L}{(1 - \eta_{kk}L)(1 - \phi L)}\varepsilon_t. \tag{31}$$

The first equality in (31) shows that technological shocks affect output both directly and indirectly through capital accumulation. The second equality shows that the sum of the two effects is an ARMA(2, 1) process for output.

The solution for consumption is obtained by substituting (29) and (30) into the expression $c_t = \eta_{ck}k_t + \eta_{ca}a_t$. This too is an ARMA(2, 1) process:

$$c_t = \frac{\eta_{ck}\eta_{ka}L}{(1 - \eta_{kk}L)(1 - \phi L)}\varepsilon_t + \frac{\eta_{ca}}{(1 - \phi L)}\varepsilon_1$$

$$= \frac{\eta_{ca} + (\eta_{ck}\eta_{ka} - \eta_{ca}\eta_{kk})L}{(1 - \eta_{kk}L)(1 - \phi L)}\varepsilon_t. \tag{32}$$

The capital, output, and consumption processes all have the same autoregressive roots η_{kk} and ϕ.[5]

2.6. Interpretation of the elasticities, and some special cases

Table 1 reports numerical values of the elasticities η_{ck}, η_{ca} and η_{kk}, η_{ka}, for the benchmark parameters discussed above and for various choices of the parameters σ and ϕ. σ is set equal to 0, 0.2, 1, 5, and ∞ to cover the whole range of possibilities. These choices for σ correspond to values for the discount factor β of ∞, 1.010, 0.990, 0.986, and 0.985, respectively, since eq. (7) implies a discount factor greater than 1 if σ is less than $g/r = 1/3$.[6] The persistence parameter ϕ is set equal to 0, 0.5, 0.95, and 1, again to cover the whole range of possibilities. Variation in ϕ has more important effects on the model when ϕ is close to 1, which is why both $\phi = 0.95$ and $\phi = 1$ are included.

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[5]These results can easily be generalized for more complicated technology processes. For example an AR(p) technology process generates an ARMA($p + 1$, $p - 1$) for the capital stock and an ARMA($p + 1$, p) for output, consumption, and the real interest rate. All these variables have common autoregressive roots.

[6]Kocherlakota (1988) argues for a small value of σ and a time discount factor greater than 1.

Table 1

Consumption and capital elasticities for the fixed-labor model with technology shocks.[a]

ϕ	σ				
	0	0.2	1	5	∞
0.00	0.11, 0.01	0.30, 0.02	0.59, 0.05	1.21, 0.10	11.30, 0.89
	1.00, 0.08	0.98, 0.08	0.96, 0.07	0.90, 0.07	0.00, 0.00
0.50	0.11, 0.02	0.30, 0.04	0.59, 0.06	1.21, 0.06	11.30, −4.69
	1.00, 0.08	0.98, 0.07	0.96, 0.07	0.90, 0.07	0.00, 0.50
0.95	0.11, 0.15	0.30, 0.25	0.59, 0.23	1.21, −0.12	11.30, −9.70
	1.00, 0.07	0.98, 0.06	0.96, 0.06	0.90, 0.09	0.00, 0.95
1.00	0.11, 0.89	0.30, 0.70	0.59, 0.41	1.21, −0.21	11.30, −10.30
	1.00, 0.00	0.98, 0.02	0.96, 0.04	0.90, 0.10	0.00, 1.00

[a]σ is the intertemporal elasticity of substitution and ϕ is the persistence of the AR(1) technology shock. The model is specified in eqs. (11), (13), (17), and (18) in the text. The top two numbers in each group are η_{ck}, η_{ca}, where η_{ck} is the elasticity of consumption with respect to the capital stock and η_{ca} is the elasticity of consumption with respect to technology. The bottom two numbers in each group are η_{kk}, η_{ka}, where η_{kk} is the elasticity of next period's capital stock with respect to this period's capital stock and η_{ka} is the elasticity of next period's capital stock with respect to this period's technology.

Several points are worth noting. First, the coefficient η_{ck} does not depend on the persistence of technology shocks ϕ but is increasing in the elasticity of intertemporal substitution σ. To understand this, recall that η_{ck} measures the effect on current consumption of an increase in capital with a fixed level of technology. Such an increase has a positive income effect on current consumption that does not depend on the value of σ. It also lowers the real interest rate, creating a positive substitution effect on current consumption that is stronger the greater the parameter σ.

Second, the coefficient η_{kk} also does not depend on ϕ but declines with σ. This follows from the fact that $\eta_{kk} = \lambda_1 + (1 - \lambda_1 - \lambda_2)\eta_{ck}$. In a model with non-stochastic technology, $1 - \eta_{kk}$ measures the rate of convergence to steady state as studied by Barro and Sala-i-Martin (1992) among others. Barro and Sala-i-Martin find that empirically $1 - \eta_{kk}$ (which they call β) equals about 0.02 at an annual rate or 0.005 at a quarterly rate. Table 1 shows that $1 - \eta_{kk}$ can be this small with the benchmark parameter values if the elasticity of intertemporal substitution σ is very small (between 0 and 0.2). Barro and Sala-i-Martin mention this possibility, but emphasize instead the fact that a smaller labor share α (corresponding to a broader concept of capital) can reduce the convergence rate.

Third, the coefficient η_{ca} is increasing in persistence ϕ for low values of σ, but decreasing for high values of σ. To understand this, recall that η_{ca} measures the effect on current consumption of an increase in technology with a fixed stock of

capital. At low values of σ, substitution effects are weak and the agent responds primarily to income effects. A technology shock has an income effect which is stronger when the shock is more persistent, hence η_{ca} increases with ϕ. At high values of σ, substitution effects are important. A purely temporary technology shock ($\phi = 0$) does not directly affect the real interest rate; it is like a windfall gain in current output. The agent is deterred from saving this windfall by the increase in the capital stock and reduction in the interest rate that would result, hence η_{ca} is large. A persistent technology shock, on the other hand, increases the real interest rate today and in the future. This encourages saving, making η_{ca} small or even negative.

It is worth discussing explicitly some special cases of the general model. The case $\phi = 1$, in which log technology follows a random walk, is often assumed in the literature [Christiano and Eichenbaum (1992), King, Plosser, Stock, and Watson (1991), Prescott (1986)]. In this case the model solution has the property that $\eta_{ck} + \eta_{ca} = 1$ and $\eta_{kk} + \eta_{ka} = 1$. One can then show that although log technology, capital, output, and consumption follow unit root processes, they are cointegrated because the difference between any two of them is stationary. To see this for log technology and capital, note that (32) gives the stochastic process for λ_3 times the log technology–capital ratio. When $\eta_{kk} + \eta_{ka} = 1$, the unit autoregressive root cancels with a unit moving average root and we have an AR(1) for the log technology–capital ratio with coefficient η_{kk}. The real interest rate, of course, follows the same process.

Another interesting special case has $\sigma = \infty$ or equivalently $\gamma = 0$, so that the representative agent is risk-neutral. In this case the model solution simplifies considerably because the quadratic coefficient Q_2 in eq. (24) becomes negligibly small relative to the other coefficients. (24) becomes a linear equation that can be solved to obtain $\eta_{ck} = -\lambda_1/(1 - \lambda_1 - \lambda_2) = 11.3$, the steady-state value of the capital–consumption ratio. Risk neutrality fixes the *ex ante* real interest rate, and hence the level of capital for a given level of technology. With fixed technology any increase in capital is simply consumed, so the derivative of consumption with respect to capital is 1 and the elasticity η_{ck} is the capital–consumption ratio. It follows that an increase in capital today does not increase capital tomorrow, so $\eta_{kk} = 0$. Finally, $\eta_{ka} = \phi$, because the capital stock changes proportionally with the level of technology. Capital is an AR(1) process with coefficient ϕ, while output and consumption are ARMA(1, 1) processes.

The opposite extreme case has $\sigma = 0$. Here intertemporal substitution is entirely absent from the model. Again the solution simplifies because the intercept $Q_0 = 0$ in the quadratic eq. (24) for η_{ck}, which therefore collapses to a linear equation. We have $\eta_{ck} = (1 - \lambda_1)/(1 - \lambda_1 - \lambda_2) = 0.11$. In this case an increase in capital, with fixed technology, stimulates only as much extra consumption as can be permanently sustained. The derivative of consumption with respect to capital is the annuity value of a unit increase in capital, $-(1 - \lambda_1)/\lambda_1 = (r - g)/(1 + r)$, and the elasticity is this derivative times the

steady-state capital–consumption ratio. It follows that a unit increase in capital today generates a unit increase in capital tomorrow, so $\eta_{kk} = 1$.

It is straightforward to show that when $\sigma = 0$ log consumption follows a random walk, while log output and log capital follow unit root processes cointegrated with log consumption. This model differs from the $\phi = 1$ case in that the stationary linear combination of log consumption and log capital is not the log ratio $c_t - k_t$, but is instead $c_t - \eta_{ck}\mu_t = c_t - 0.11\,k_t$. An increase in capital does not lead to a proportional increase in consumption in the long run, because the marginal product of capital is less than the average product. Associated with this, there are some technical difficulties with the $\sigma = 0$ model. First, eq. (7) implies that as σ approaches 0, the time discount factor must increase to infinity to maintain a finite steady-state interest rate. Second, when $\sigma = 0$ and technology is stationary ($\phi < 1$), the log technology–capital ratio is nonstationary. This invalidates the loglinear approximations used to obtain the solution. Thus strictly speaking the discussion above applies only to very small but nonzero values of σ.

Despite these problems, the stochastic growth model with $\sigma = 0$ deserves attention because it is a general equilibrium version of the permanent income model of Hall (1978) and Flavin (1981).[7] In this model temporary technology shocks cause temporary variation in output but not in consumption, so output is more variable than consumption and the consumption–output ratio forecasts changes in output. Fama (1992) advocates a model of this type, but does not provide a formal analysis. Hall (1988) and Campbell and Mankiw (1989) demonstrate the empirical relevance of the model with small σ by showing that predictable movements in real interest rates have been only weakly associated with predictable consumption growth in postwar U.S. data.[8]

The $\sigma = 0$ case also plays an interesting role in welfare analysis of the model. The maximized welfare of the representative agent can be written as a loglinear function of capital and technology by approximating Bellman's equation. I write the maximized objective function defined in (3) as $V_t^{1-\gamma}/(1-\gamma)$, so that V_t has the same units as consumption. The loglinear approximation of Bellman's equation (derived in appendix A) is then

$$(1 - \lambda_1)(c_t - v_t) = \mathrm{E}_t v_{t+1} - v_t. \tag{33}$$

This equation implies that v_t can be written as an expected discounted value of future log consumption, where the discount factor is $1/\lambda_1 = 0.99$ at benchmark

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[7]Christiano, Eichenbaum, and Marshall (1991) and Hansen (1987) present an alternative general equilibrium permanent income model in which there is a linear storage technology which fixes the real interest rate. Here the real interest rate varies but consumers ignore this when $\sigma = 0$ because they are infinitely averse to intertemporal substitution.

[8]Campbell and Mankiw also argue that there is a predictable component of consumption growth correlated with predictable income growth, a phenomenon not modelled here.

parameter values. The solution for v_t takes the form $v_t = \eta_{vk} k_t + \eta_{va} a_t$. For any parameter values $\eta_{vk} = (1 - \lambda_1)/(1 - \lambda_1 - \lambda_2) = 0.11$, the value of η_{ck} in the $\sigma = 0$ case. The elasticity with respect to technology, η_{va}, varies with the persistence parameter ϕ but not with the intertemporal elasticity of substitution σ. For any σ, η_{va} is always equal to the value of η_{ca} in the $\sigma = 0$ case.

The interpretation of these results is straightforward. A 1% increase in capital increases the welfare of the representative agent by the same amount as an $\eta_{vk} = 0.11\%$ permanent increase in consumption. η_{vk} does not depend on the parameters of the agent's utility function, and it can be measured by looking at the permanent consumption increase that the agent optimally chooses in the $\sigma = 0$ case. Similarly, a 1% increase in technology has the same welfare effect as an $\eta_{va}\%$ permanent increase in consumption. η_{va} can be found by looking at the permanent consumption increase chosen in the $\sigma = 0$ case. A 1% temporary increase in technology has a welfare effect equivalent to a 0.01% permanent increase in consumption, while a 1% permanent increase in technology has a much larger welfare effect equivalent to a 0.89% permanent increase in consumption.

2.7. Longer-run dynamics, and more general technology processes

Figs. 1, 2, and 3 illustrate the consequences of alternative parameter values for the dynamic response of output to technology shocks. In each case the initial response of output to a unit technology shock is just $\alpha = 0.667$, the exponent on technology in the production function. Fig. 1 shows responses to a technology shock with persistence $\phi = 0.5$. The different response lines correspond to the five values of σ studied in table 1. None of the responses are very different from the underlying AR(1) technology shock itself, because a transitory technology shock does not generate sufficient capital accumulation to have an important effect on output. To the extent that there is variation across σ values, higher values give higher output initially but lower output in the long run. The reason is that an agent with a high value of σ accumulates capital aggressively in response to the initial technology shock and then decumulates it rapidly when the technology shock disappears. An agent with a low value of σ, on the other hand, accumulates less capital but holds onto capital longer. In the extreme case $\sigma = 0$, capital and output are permanently higher in the wake of a temporary technology shock.

Figs. 2 and 3 show output responses to technology shocks with persistence $\phi = 0.95$ and $\phi = 1$, respectively. Fig. 2 is similar to fig. 1, except that the different lines are further apart and output has a hump-shaped impulse response when σ is sufficiently high. Capital accumulation can now make the medium-run output response higher than the short-run response. In fig. 3 the long-run output response is one for any positive value of σ, because of the cointegration property of the $\phi = 1$ model discussed above. The speed of adjustment to the long run is
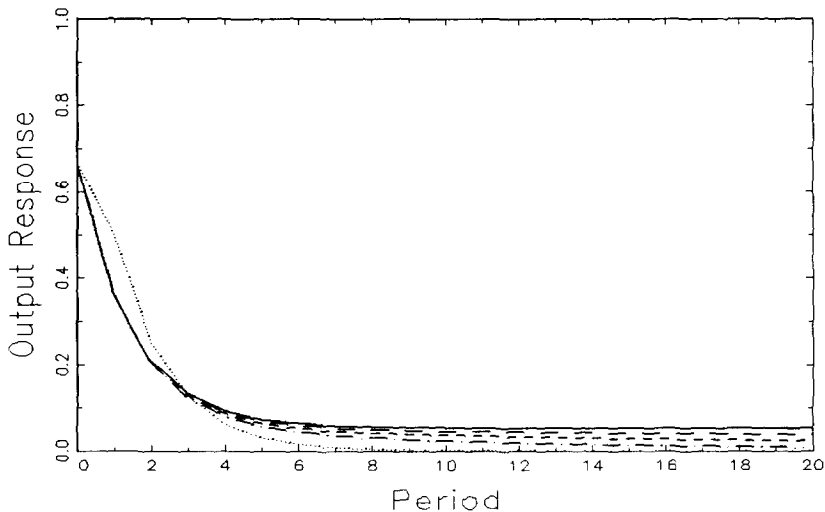
Fig. 1. Output response to a technology shock with fixed labor supply and $\phi = 0.5$.

The solid line gives the percentage response of output to a 1% technology shock in a model with fixed labor supply, specified in eqs. (11), (13), (17), and (18), when the intertemporal elasticity of substitution $\sigma = 0$. The long-dashed line gives the response when $\sigma = 0.2$. The short-dashed line gives the response when $\sigma = 1$. The dashed and dotted line gives the response when $\sigma = 5$. The dotted line gives the response when $\sigma = \infty$. In all cases initial response is $\alpha = 0.667$.

governed by σ, which determines η_{ck} and hence the convergence parameter η_{kk}. As already discussed, convergence is more rapid when σ is larger; in the extreme case of infinite σ, the adjustment takes place in one period.

An important feature of the loglinear model is that the solutions for simple AR(1) technology shocks can be combined to obtain solutions for more complicated technology processes. Suppose that log technology a_t is the sum of two components a_{1t} and a_{2t}, each of which follows an AR(1) and is observed by the representative agent. It is straightforward to show that any endogenous variable z_t obeys $z_t = \eta_{zk} k_t + \eta_{z1} a_{1t} + \eta_{z2} a_{2t}$, where η_{z1} is the solution already obtained for η_{za} when log technology equals a_{1t}, and η_{z2} is the solution for η_{za} when log technology equals a_{2t}. This result generalizes in the obvious way to any number of separately observed components, which may have arbitrary correlations.

As an empirically relevant example, suppose that a_{1t} and a_{2t} have persistence parameters 0.95 and 1, respectively, and that their innovations have the same variance and are perfectly negatively correlated. Then a unit technology shock consists of a positive shock that decays at rate 0.95, combined with a negative permanent shock. Such a shock causes technology (measured relative to its previous steady-state growth path) to decline gradually to a new, permanently
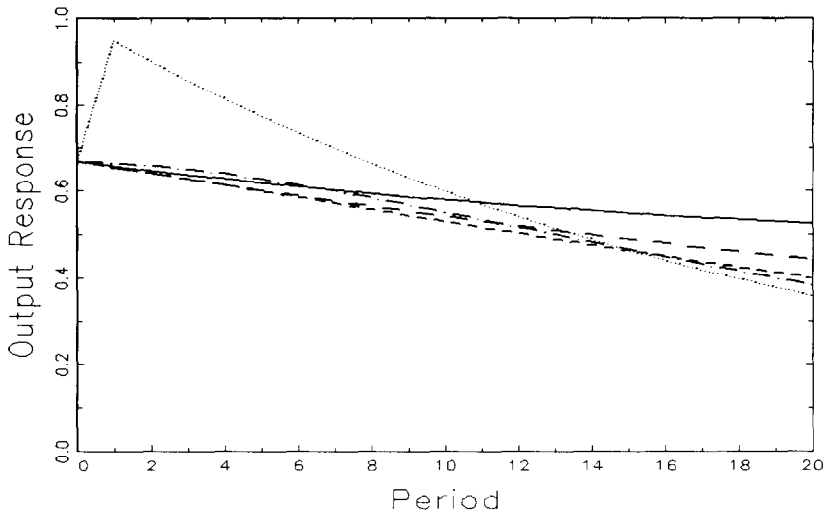
Fig. 2. Output response to a technology shock with fixed labor supply and $\phi = 0.95$.

The solid line gives the percentage response of output to a 1% technology shock in a model with fixed labor supply, specified in eqs. (11), (13), (17), and (18), when the intertemporal elasticity of substitution $\sigma = 0$. The long-dashed line gives the response when $\sigma = 0.2$. The short-dashed line gives the response when $\sigma = 1$. The dashed and dotted line gives the response when $\sigma = 5$. The dotted line gives the response when $\sigma = \infty$. In all cases the initial response is $\alpha = 0.667$.

lower level. It therefore approximates a 'productivity slowdown' of the type experienced in the U.S. in the 1970's.

Fig. 4 illustrates the effects of such a shock on output, consumption, and capital over a ten-year period. The figure assumes that $\sigma = 1$. Technology is represented by a dotted line declining geometrically towards its new permanent level 1% below the old permanent level. The half-life of the technology decline is just over three years and almost 90% of the decline is completed after ten years. The long-dashed line represents consumption. Because the technology decline is anticipated, permanent income considerations immediately reduce consumption by about 0.8%. This initially leads to capital accumulation, as shown by the short-dashed line for the capital stock. In less than two years, however, the capital stock starts to decline towards its new steady-state level. Because capital is high relative to technology during the transition to the new steady state, output (shown by a solid line) is also high relative to technology.

It is sometimes argued on permanent income grounds that a productivity slowdown should unambiguously increase saving. It is true that throughout the transition shown in the figure for the $\sigma = 1$ case, consumption is unusually low relative to output. However this corresponds to faster capital accumulation only for the first two years. After that, capital is decumulated despite the low
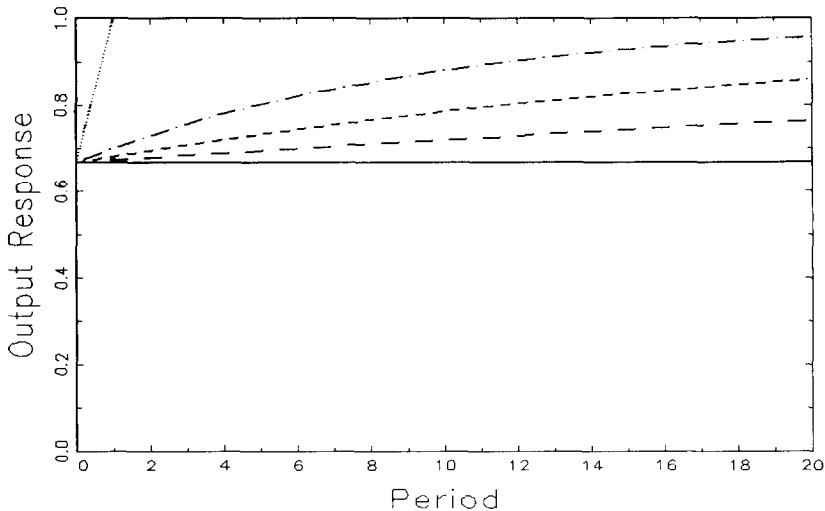
Fig. 3. Output response to a technology shock with fixed labor supply and $\phi = 1$.

The solid line gives the percentage response of output to a 1% technology shock in a model with fixed labor supply, specified in eqs. (11), (13), (17), and (18), when the intertemporal elasticity of substitution $\sigma = 0$. The long-dashed line gives the response when $\sigma = 0.2$. The short-dashed line gives the response when $\sigma = 1$. The dashed and dotted line gives the response when $\sigma = 5$. The dotted line gives the response when $\sigma = \infty$. In all cases the initial response is $\alpha = 0.667$.

consumption–output ratio because output is low relative to capital. This decumulation must occur (for any strictly positive σ), so that the economy can reach its new steady-state growth path with the same ratio of capital to technology that it had on the old growth path. Furthermore, if the elasticity of intertemporal substitution is large enough, consumption can actually rise relative to output at the onset of a productivity slowdown. This occurs for any value of σ such that η_{ca} declines with persistence ϕ. Table 1 shows that an elasticity of intertemporal substitution of 5 is already large enough to produce this behavior.

2.8. Summary

Before moving on to the variable-labor model, three characteristics of the fixed-labor model deserve particular note. First, the impulse responses plotted in figs. 1, 2, and 3 show that capital accumulation has an important effect on the dynamics of the economy only when the underlying technology shock is persistent, lasting long enough for significant changes in capital to occur. The stochastic growth model is unable to generate persistent effects from transitory shocks.[9]

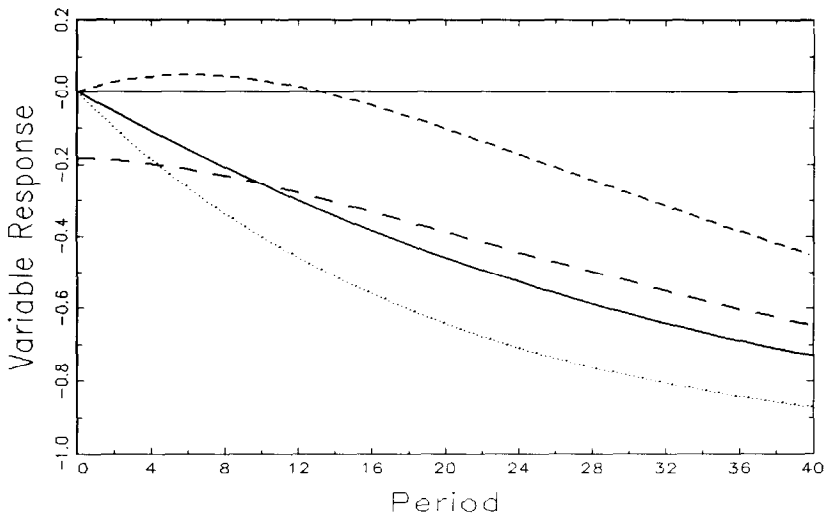[9]Blanchard and Fischer (1989) emphasize this point.

Fig. 4. Response of the economy to a productivity slowdown with fixed labor supply.

This figure shows the percentage responses of several variables to a 1% permanent negative decline in technology, accompanied by a 1% transitory increase in technology with persistence $\phi = 0.95$. The dotted line gives the implied path of technology. The responses of other variables are calculated in a model with fixed labor supply and intertemporal elasticity of substitution σ equal to 1. The model is specified in eqs. (11), (13), (17), and (18) in the text. The long-dashed line gives the response of consumption, the short-dashed line gives the response of the capital stock, and the solid line gives the response of output.

Second, technology shocks do not have strong effects on realized or expected returns on capital. The reason is that the gross rate of return on capital largely consists of undepreciated capital rather than the net output that is affected by technology shocks. The realized return on capital equals λ_3 times the log technology–capital ratio, and $\lambda_3 = 0.03$ at benchmark parameter values. Thus a 1% technology shock changes the realized return on capital by only three basis points, or twelve basis points at an annual rate. The expected return on capital is even more stable (and literally constant when the representative agent is risk-neutral) because capital accumulation lowers the marginal product of capital one period after a positive technology shock occurs, partially offsetting any persistent effects of the shock.

Third, capital accumulation does not generate a short- or long-run 'multiplier' in the sense of an output response to a technology shock that is larger (in percentage terms) than the underlying shock itself. None of the output responses shown in figs. 1, 2, or 3 exceed 1. This means that slower-than-normal technology growth can generate only slower-than-normal output growth and not actual declines in output. The model with fixed labor supply can explain

output declines only by appealing to implausible declines in the level of technology.

3. Variable labor supply

I now consider two models with variable labor supply. These models leave the production function (1) unchanged, but allow labor input N_t to be variable rather than constant and normalized to one. The capital accumulation eq. (2) is also unchanged. However the objective function (3) now has a period utility function involving both consumption and leisure. The first model assumes that period utility is additively separable in consumption and leisure, while the second model has nonseparable period utility.

3.1. An additively separable model

In the first model, the representative agent has log utility for consumption and power utility for leisure:

$$U(C_t, 1 - N_t) = \log(C_t) + \theta \frac{(1 - N_t)^{1 - \gamma_n}}{1 - \gamma_n}. \tag{34}$$

King, Plosser, and Rebelo (1988a) show that log utility for consumption is required to obtain constant steady-state labor supply (balanced growth) in a model with utility additively separable over consumption and leisure. The form of the utility function for leisure is not restricted by the balanced growth requirement. I use power utility for convenience and because it nests two popular special cases in the real business cycle literature: log utility for leisure in a model with divisible labor and linear derived utility for leisure in a model with indivisible labor in which workers choose lotteries over hours worked rather than choosing hours worked directly [Hansen (1985), Rogerson (1988)]. The former case has $\gamma_n = 1$ and the latter has $\gamma_n = 0$. Christiano and Eichenbaum (1992) and King, Plosser, and Rebelo (1988a) explicitly compare these two special cases. By analogy with the notation of the previous section, I define $\sigma_n \equiv 1/\gamma_n$, the elasticity of intertemporal substitution for leisure.

The first-order condition for intertemporal consumption choice remains the same as before, except that the gross marginal product of capital now depends on labor input as well as technology and the capital stock. Eq. (5) is unchanged, but (4) becomes

$$R_{t+1} \equiv (1 - \alpha)\left(\frac{A_{t+1} N_{t+1}}{K_{t+1}}\right)^{\alpha} + (1 - \delta). \tag{35}$$

The new feature of the variable-labor model is that there is now a static first-order condition for optimal choice of leisure relative to consumption at a particular date:

$$\theta(1 - N_t)^{-\gamma_n} = \frac{W_t}{C_t} = \alpha \frac{A_t^\alpha}{C_t}\left(\frac{K_t}{N_t}\right)^{1-\alpha}. \tag{36}$$

The marginal utility of leisure is set equal to the wage W_t times the marginal utility of consumption. With log utility for consumption, this is just the wage divided by consumption. The wage in turn equals the marginal product of labor from the production function (1).

Analysis of the steady state from the previous section carries over directly to the variable-labor model. The relation (7) between g and r, and the steady-state values of the ratios A_t/K_t, Y_t/K_t, and C_t/Y_t are all the same as before.

3.2. Fluctuations with separable utility

Much of the analysis of fluctuations also carries over directly from the fixed-labor-supply model. The loglinear version of the capital accumulation eq. (13) becomes

$$k_{t+1} \approx \lambda_1 k_t + \lambda_2(a_t + n_t) + (1 - \lambda_1 - \lambda_2)c_t, \tag{37}$$

where λ_1 and λ_2 are the same as before. (37) differs from (13) only in that λ_2 multiplies n_t as well as a_t. The interest rate is now $r_{t+1} = \lambda_3(a_{t+1} + n_{t+1} - k_{t+1})$, and the loglinear version of the intertemporal first-order condition (17) becomes

$$E_t\Delta c_{t+1} = \lambda_3 E_t(a_{t+1} + n_{t+1} - k_{t+1}). \tag{38}$$

Eq. (38) differs from (17) only in that σ is now equal to 1 and n_{t+1} appears in the equation. The technology shock process (18) also remains the same as before:

$$a_t = \phi a_{t-1} + \varepsilon_t. \tag{39}$$

These expressions contain an extra variable n_t, so to close the model one needs an extra equation which is provided by the static first-order condition (36). Loglinearizing in standard fashion (details are given in appendix A), I find that

$$n_t = \left(\frac{1 - N}{N}\right)\sigma_n[\alpha a_t + (1 - \alpha)(k_t - n_t) - c_t], \tag{40}$$

where N is the mean of labor supply. If, as Prescott (1986) asserts, households allocate one-third of their time to market activities, then N is $\frac{1}{3}$ and $(1 - N)/N = 2$. I shall take this as a benchmark value.

It will be convenient to rewrite (40) to express labor supply in terms of capital, technology, and consumption:

$$n_t = v[(1\text{-}\alpha)k_t + \alpha a_t - c_t], \tag{41}$$

where

$$v = v(\sigma_n) = \frac{(1 - N)\sigma_n}{N + (1 - \alpha)(1 - N)\sigma_n}. \tag{42}$$

The coefficient v is a function of σ_n. It measures the responsiveness of labor supply to shocks that change the real wage or consumption, taking into account the fact that as labor supply increases the real wage is driven down. Thus, even when utility for leisure is linear ($\sigma_n = \infty$), the coefficient v is not infinitely large. Instead, $v = 1/(1 - \alpha) = 3$ in this case. As the curvature of the utility function for leisure increases, v falls and becomes 0 when γ_n is infinite. This corresponds to the fixed-labor case studied in the previous section. Note that the value assumed for N affects only the relationship between σ_n and v, and not any other aspect of the model.

Eq. (41) can be used to substitute n_t out of eqs. (37), (38), and (39). The system is then in the same form as before, and can be solved using the same approach. Once again log consumption is linear in log capital and log technology, with coefficients η_{ck} and η_{ca}. The coefficient η_{ck} solves the quadratic eq. (24), where the coefficients Q_2, Q_1, and Q_0 are more complicated than before and are given in appendix B. The solution for η_{ca} can be obtained straightforwardly from η_{ck} and the other parameters of the model. These solutions are the same as in the previous section when labor supply is completely inelastic so that $v = 0$.

3.3. Dynamic behavior of the economy

The dynamics of the economy take the same form as in the fixed-labor model. Once again the log capital stock is a linear function of the first lags of log capital and log technology $k_{t+1} = \eta_{kk}k_t + \eta_{ka}a_t$. But now the coefficients η_{kk} and η_{ka} are given by

$$\eta_{kk} \equiv \lambda_1 + \lambda_2(1 - \alpha)v + \eta_{ck}[1 - \lambda_1 - \lambda_2(1 + v)],$$

$$\tag{43}$$

$$\eta_{ka} \equiv \lambda_2(1 + \alpha v) + \eta_{ca}[1 - \lambda_1 - \lambda_2(1 + v)].$$

Log labor supply can also be written as a linear function of log capital and technology. Substituting the expression for consumption into (41), log labor supply is

$$n_t = v(1 - \alpha - \eta_{ck})k_t + v(\alpha - \eta_{ca})a_t$$

$$\equiv \eta_{nk}k_t + \eta_{na}a_t. \tag{44}$$

Increases in capital raise the real wage by a factor $(1 - \alpha)$; this stimulates labor supply, but capital also increases consumption by a factor η_{ck}, and this can have an offsetting effect. Similarly, increases in technology raise the real wage by a factor α, but the stimulating effect on labor supply is offset by the effect η_{ca} of technology on consumption. I use the notation η_{nk} and η_{na} for the overall effects of capital and technology on labor supply.

Finally, log output can also be written as a linear function of log capital and technology:

$$y_t = [(1 - \alpha) + \alpha v(1 - \alpha - \eta_{ck})]k_t + [\alpha + \alpha v(\alpha - \eta_{ca})]a_t$$

$$\equiv \eta_{yk}k_t + \eta_{ya}a_t. \tag{45}$$

As before, this is an ARMA(2, 1) process. However, capital and technology now affect output both directly (with coefficients $1 - \alpha$ and α, respectively) and indirectly through labor supply. The initial response to a technology shock is now $\alpha + \alpha v(\alpha - \eta_{ca})$ rather than α. Thus, the variable-labor model can produce an amplified output response to technology shocks, even in the very short run.

Tables 2 and 3 illustrate the solution of the model for the same values of σ_n and ϕ that were used for σ and ϕ in table 1. Table 2 shows the consumption and capital elasticities that were reported in table 1; table 3 gives employment and output elasticities.

When $\sigma_n = 0$ (the first column of tables 2 and 3), the model is the same as the model with fixed labor supply and log utility over consumption (the third column of table 1). In this case the coefficients η_{nk} and η_{na} are both 0. As σ_n increases, the coefficient η_{nk} becomes increasingly negative, while η_{na} becomes increasingly positive. Thus, an increase in capital lowers work effort because it increases consumption more than it increases the real wage. A positive technology shock increases work effort. The coefficient η_{nk} is independent of the persistence of technology ϕ, but the coefficient η_{na} declines with ϕ. The reason is that a persistent technology shock increases consumption more than a transitory one does (this is shown by the fact that η_{ca} increases with ϕ in the table). The increase in consumption lowers the marginal utility of income and reduces work effort. Put another way, transitory technology shocks produce a stronger intertemporal substitution effect in labor supply.

Table 2

Consumption and capital elasticities for the separable variable-labor model with technology shocks.[a]

	σ_n				
ϕ	0	0.2	1	5	∞
0.00	0.59, 0.05	0.57, 0.05	0.54, 0.07	0.51, 0.10	0.50, 0.11
	0.96, 0.08	0.95, 0.09	0.94, 0.13	0.93, 0.18	0.93, 0.20
0.50	0.59, 0.06	0.57, 0.08	0.54, 0.10	0.51, 0.12	0.50, 0.14
	0.96, 0.07	0.95, 0.09	0.94, 0.13	0.93, 0.17	0.93, 0.19
0.95	0.59, 0.23	0.57, 0.25	0.54, 0.29	0.51, 0.33	0.50, 0.35
	0.96, 0.06	0.95, 0.07	0.94, 0.09	0.93, 0.11	0.93, 0.12
1.00	0.59, 0.41	0.57, 0.43	0.54, 0.46	0.51, 0.49	0.50, 0.50
	0.96, 0.04	0.95, 0.05	0.94, 0.06	0.93, 0.07	0.93, 0.07

[a] σ_n is the elasticity of labor supply and ϕ is the persistence of the AR(1) technology shock. The model is specified in eqs. (34)–(42) in the text. The top two numbers in each group are η_{ck}, η_{ca}, where η_{ck} is the elasticity of consumption with respect to the capital stock and η_{ca} is the elasticity of consumption with respect to technology. The bottom two numbers in each group are η_{kk}, η_{ka}, where η_{kk} is the elasticity of next period's capital stock with respect to this period's capital stock and η_{ka} is the elasticity of next period's capital stock with respect to this period's technology.

Once again several special cases of the model are worth attention. The random walk model for log technology ($\phi = 1$) is cointegrated just as in the fixed-labor model. Again we have $\eta_{ck} + \eta_{ca} = 1$ and $\eta_{kk} + \eta_{ka} = 1$, but also $\eta_{nk} + \eta_{na} = 0$. In this model labor supply effects are quite weak: Even with linear utility for leisure (an infinite σ_n), a 1% increase in technology stimulates only a 0.49% increase in work effort. As mentioned above, the case $\sigma_n = 0$ is the fixed-labor model with log utility for consumption. The opposite extreme case $\sigma_n = \infty$ solves relatively easily because $(1 - \alpha)v = 1$, so the intercept term Q_0 in the equation for η_{ck} is 0 and this equation becomes linear. However, the coefficients obtained in this case do not have any straightforward interpretation.

Tables 2 and 3 can also be used to calculate the elasticities of gross factor returns with respect to capital and technology. The return on capital $r_{t+1} = \lambda_3(a_{t+1} + n_{t+1} - k_{t+1})$, so its response to capital is $\lambda_3(\eta_{nk} - 1)$ and its response to technology is $\lambda_3(1 + \eta_{na})$. These responses are small, just as they were in the fixed-labor model. The largest possible effect of a 1% technology shock on the realized return on capital is eight basis points at a quarterly rate, or 32 basis points at an annual rate, when the technology shock is purely temporary and the utility function is linear in leisure. The expected return on capital is also stable for similar reasons.

The log real wage rate equals $y_t - n_t$, so its response to capital is $\eta_{yk} - \eta_{nk}$ and its response to technology is $\eta_{ya} - \eta_{na}$. Inspection of table 3 shows that the

Table 3

Employment and output elasticities for the separable variable-labor model with technology shocks.[a]

| | σ_n | | | | |
ϕ	0	0.2	1	5	∞
0.00	0.00, 0.00	− 0.08, 0.22	− 0.24, 0.71	− 0.40, 1.32	− 0.49, 1.67
	0.33, 0.67	0.28, 0.81	0.17, 1.14	0.06, 1.54	0.01, 1.78
0.50	0.00, 0.00	− 0.08, 0.21	− 0.24, 0.68	− 0.40, 1.25	− 0.49, 1.58
	0.33, 0.67	0.28, 0.81	0.17, 1.12	0.06, 1.50	0.01, 1.72
0.95	0.00, 0.00	− 0.08, 0.15	− 0.24, 0.45	− 0.40, 0.78	− 0.49, 0.95
	0.33, 0.67	0.28, 0.77	0.17, 0.97	0.06, 1.18	0.01, 1.30
1.00	0.00, 0.00	− 0.08, 0.08	− 0.24, 0.24	− 0.40, 0.40	− 0.49, 0.49
	0.33, 0.67	0.28, 0.72	0.17, 0.83	0.06, 0.94	0.01, 0.99

[a]σ_n is the elasticity of labor supply and ϕ is the persistence of the AR(1) technology shock. The model is specified in eqs. (34)–(42) in the text. The top two numbers in each group are η_{nk}, η_{na}, where η_{nk} is the elasticity of employment with respect to the capital stock and η_{na} is the elasticity of employment with respect to technology. The bottom two numbers in each group are η_{yk}, η_{ya}, where η_{yk} is the elasticity of output with respect to the capital stock and η_{ya} is the elasticity of output with respect to technology.

elasticity of the wage with respect to technology is smallest when utility is linear in leisure. In this case (the right-hand column of table 3) the real wage elasticity is the same as the consumption elasticity η_{ca}, because linear utility in leisure fixes the wage–consumption ratio. Depending on its persistence, a 1% technology shock can raise the real wage by 0.11% to 0.50%. Somewhat greater real wage effects are obtained when labor supply is inelastic. In the extreme fixed-labor case (the left-hand column of table 3), a 1% transitory or persistent technology shock raises the real wage by 0.67%. As Christiano and Eichenbaum (1992) emphasize, in this model the marginal product of labor is proportional to the average product, so elasticities for labor productivity are the same as those for the real wage.

Variable labor supply has important implications for the short-run elasticity of output with respect to technology, η_{ya}. Recall that when labor supply is fixed ($v = 0$), this elasticity is just $\alpha = 0.667$. With variable labor supply, $\eta_{ya} = \alpha + \alpha v(\alpha - \eta_{ca})$. This can exceed 1, reaching a maximum of 1.78 when $v = 3$ and $\phi = 0$. The elasticity falls with ϕ, however, and when $\phi = 1$, it cannot exceed 0.99. This is important because an elasticity greater than 1 allows absolute declines in output to be generated by positive but slower-than-normal growth in technology; this is surely more plausible than the notion that there are absolute declines in technology. The elasticity is illustrated in fig. 5, a contour plot of η_{ya} against the parameters v and ϕ.
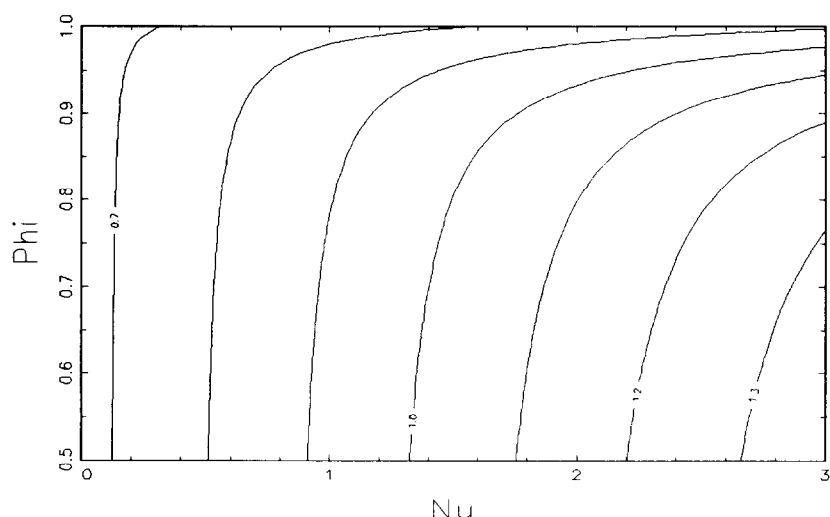
Fig. 5. Initial output response to a technology shock with variable labor supply and separable utility.

The contours show the elasticity of output with respect to technology in a model with variable labor supply and additively separable utility over consumption and leisure. The model is specified in eqs. (34)–(42) in the text. The elasticity is plotted for different values of the parameters v and ϕ, where v is a function of the elasticity of labor supply defined in eq. (42) and ϕ is the persistence of technology shocks. The contour lines are 0.1 apart. Note that the smallest value of ϕ shown is 0.5, and that when $v = 0$, the elasticity is $\alpha = 0.667$ for any value of ϕ.

3.4. A nonseparable model

An alternative specification that is consistent with balanced growth is the nonadditively separable Cobb–Douglas utility function,

$$U(C_t, N_t) = [C_t^\rho (1 - N_t)^{1-\rho}]^{1-\gamma}/(1 - \gamma). \tag{46}$$

This is used by Eichenbaum, Hansen, and Singleton (1988) and Prescott (1986). When $\gamma = \sigma = 1$, this utility function is the same as the additively separable utility function with $\sigma_n = 1$.

The steady state for this model is similar to that for the previous model. The steady-state output–capital and output–consumption ratios are the same as before, but the equation relating the growth rate, the utility discount rate, and the interest rate is slightly altered from (7) to

$$g = \frac{\log(\beta) + r}{1 - \rho(1 - \gamma)}. \tag{47}$$

The parameter ρ determines the fraction of time devoted to market activities, N. Given N one can calculate the implied ρ as $\rho = 1/(1 + [(1 - N)\alpha(Y/C)/N])$, where Y/C is the steady-state output–consumption ratio. At the benchmark parameter values, with $N = 0.33$, $\rho = 0.36$.

The approximate loglinear model of fluctuations has the same capital accumulation equation as before. The static first-order condition for optimal labor supply does not depend on the curvature of the utility function and is

$$n_t = v(1)[(1 - \alpha)k_t + \alpha a_t - c_t], \tag{48}$$

where $v(1)$ is given by (42) setting $\sigma_n = 1$. The intertemporal first-order condition is somewhat more complicated than in the separable case. It takes the form

$$[1 - \rho(1 - \gamma)]\mathrm{E}_t\Delta c_{t+1} + (1 - \rho)(1 - \gamma)\left(\frac{N}{1 - N}\right)\mathrm{E}_t\Delta n_{t+1}$$

$$= \lambda_3 \mathrm{E}_t(a_{t+1} + n_{t+1} - k_{t+1}). \tag{49}$$

As γ increases, the representative agent becomes more averse to intertemporal substitution. In the limit with an infinite γ and $\sigma = 0$, eq. (49) implies $\mathrm{E}_t\Delta c_{t+1} = [(1 - \rho)N/\rho(1 - N)]\mathrm{E}_t\Delta n_{t+1} = 0.88\Delta n_{t+1}$ at benchmark parameter values. In this case the representative agent's marginal utility follows

Table 4

Consumption and capital elasticities for the nonseparable variable-labor model with technology shocks.[a]

ϕ	σ				
	0	0.2	1	5	∞
0.00	0.23, 0.35	0.37, 0.28	0.54, 0.07	0.71, −0.30	0.82, −0.62
	1.00, 0.08	0.97, 0.09	0.94, 0.13	0.91, 0.20	0.89, 0.26
0.50	0.23, 0.35	0.37, 0.29	0.54, 0.10	0.71, −0.24	0.82, −0.53
	1.00, 0.08	0.97, 0.09	0.94, 0.13	0.91, 0.19	0.89, 0.24
0.95	0.23, 0.42	0.37, 0.42	0.54, 0.29	0.71, 0.09	0.82, −0.06
	1.00, 0.07	0.97, 0.07	0.94, 0.09	0.91, 0.45	0.89, 0.16
1.00	0.23, 0.77	0.37, 0.63	0.54, 0.46	0.71, 0.29	0.82, 0.18
	1.00, 0.00	0.97, 0.03	0.94, 0.06	0.91, 0.09	0.89, 0.11

[a] σ is the elasticity of intertemporal substitution and ϕ is the persistence of the AR(1) technology shock. The model is specified in eqs. (46)–(49) in the text. The top two numbers in each group are η_{ck}, η_{ca}, where η_{ck} is the elasticity of consumption with respect to the capital stock and η_{ca} is the elasticity of consumption with respect to technology. The bottom two numbers in each group are η_{kk}, η_{ka}, where η_{kk} is the elasticity of next period's capital stock with respect to this period's capital stock and η_{ka} is the elasticity of next period's capital stock with respect to this period's technology.

Table 5

Employment and output elasticities for the nonseparable variable-labor model with technology shocks.[a]

ϕ	σ				
	0	0.2	1	5	∞
0.00	0.13, 0.38	-0.05, 0.46	-0.24, 0.71	-0.45, 1.16	-0.58, 1.54
	0.42, 0.92	0.30, 0.98	0.17, 1.14	0.03, 1.44	-0.05, 1.69
0.50	0.13, 0.38	-0.05, 0.45	-0.24, 0.68	-0.45, 1.09	-0.58, 1.44
	0.42, 0.92	0.30, 0.97	0.17, 1.12	0.03, 1.40	-0.05, 1.63
0.95	0.13, 0.30	-0.05, 0.29	-0.24, 0.45	-0.45, 0.70	-0.58, 0.88
	0.42, 0.87	0.28, 0.86	0.17, 0.97	0.03, 1.13	-0.05, 1.25
1.00	0.13, -0.13	-0.05, 0.05	-0.24, 0.24	-0.45, 0.45	-0.58, 0.58
	0.42, 0.58	0.30, 0.70	0.17, 0.83	0.03, 0.97	-0.05, 1.05

[a] σ is the elasticity of intertemporal substitution and ϕ is the persistence of the AR(1) technology shock. The model is specified in eqs. (46)–(49) in the text. The top two numbers in each group are η_{nk}, η_{na}, where η_{nk} is the elasticity of employment with respect to the capital stock and η_{na} is the elasticity of employment with respect to technology. The bottom two numbers in each group are η_{yk}, η_{ya}, where η_{yk} is the elasticity of output with respect to the capital stock and η_{ya} is the elasticity of output with respect to technology.

a random walk, but neither log consumption nor log labor supply need follow random walks because of the nonseparability in utility.

Solution of the nonseparable model proceeds in standard fashion, described explicitly in appendix B. Consumption and capital elasticities for this model are given in table 4 and employment and output elasticities are given in table 5. Comparing table 4 with table 2, the nonseparable model allows a much wider range of consumption elasticities because it does not fix the curvature of the utility function. However, this does not have a major effect on output elasticities. Comparing table 5 with table 3, the output response to technology shocks covers roughly the same range in the nonseparable model as it did in the separable model. The largest possible response to a temporary technology shock is slightly smaller in the nonseparable model, but the largest possible response to a permanent shock is slightly larger. This means that the nonseparable model can produce a multiplier slightly greater than 1 even when technology shocks are permanent.

3.5. The productivity slowdown with variable labor

Just as in the fixed-labor model, the solutions obtained above can be combined to describe responses to more general technology processes. Fig. 6 shows the response of the economy to a productivity slowdown (a positive shock with
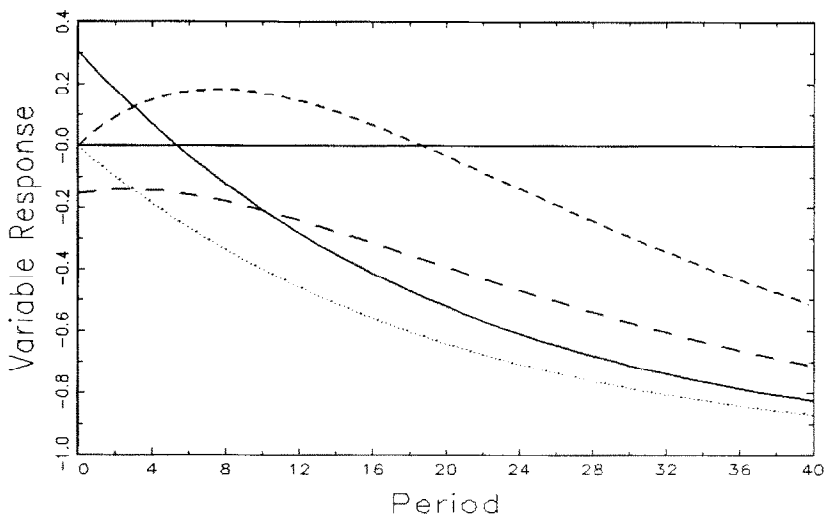
Fig. 6. Response of the economy to a productivity slowdown with variable labor supply and separable utility.

This figure shows the percentage responses of several variables to a 1% permanent negative decline in technology, accompanied by a 1% transitory increase in technology with persistence $\phi = 0.95$. The dotted line gives the implied path of technology. The responses of other variables are calculated in a model with variable labor supply and additively separable utility over consumption and leisure. The model is specified in eqs. (34)–(42) in the text. The elasticity of labor supply σ_n is assumed to equal 1. The long-dashed line gives the response of consumption, the short-dashed line gives the response of the capital stock, and the solid line gives the response of output.

persistence 0.95, combined with a negative shock with persistence 1), under the assumption of log utility for consumption and leisure. As noted above, this utility specification can be obtained from the separable model with $\sigma_n = 1$ or from the nonseparable model with $\sigma = 1$.

The dynamics shown in fig. 6 are similar to those in fig. 4. Consumption drops immediately, which leads to a period of capital accumulation before capital gradually declines to its new steady-state value. There are however two new features in fig. 6. First, in the later stages of the transition the consumption–output ratio is above its steady-state level because low real interest rates stimulate consumption. Second and more important, the initial drop in consumption is accompanied by an increase in work effort (since the technology shock has no immediate impact on the real wage, and the marginal utility of consumption is higher). This raises output initially, and leads to a more pronounced accumulation of capital than in fig. 4. Output falls below its old steady-state level one year after the initial shock, but capital does not fall below this level until four years after the shock. It is straightforward to verify from

tables 3 and 5 that this effect is robust: The initial output response to the productivity slowdown is positive for any possible value of σ or σ_n.

This example illustrates an important point. In a model with variable labor supply, the responses of employment and output to a technology shock decline with the persistence of that shock. If the shock is more persistent than a random walk, so that its ultimate effect is larger than its initial effect, then it is possible to get a perverse initial response of employment and output. The reason is that a highly persistent shock has a large initial effect on the marginal utility of consumption relative to its initial effect on the real wage.

4. Government spending and taxation

The stochastic growth model can also be subjected to other types of shocks. In this section I study the effects of government spending. For simplicity I assume throughout that government spending does not enter the production function or the utility function of the representative agent. The effects of government spending depend critically on the assumed tax system [Baxter and King (1993)]; here I first study lump-sum taxes and then consider a simple form of distortionary income taxation.

4.1. Lump-sum taxation

When government spending is financed by lump-sum taxation, all first-order conditions are the same as before. Only the capital accumulation equation changes, becoming

$$K_{t+1} = (1 - \delta)K_t + Y_t - C_t - X_t, \tag{50}$$

where X_t is the level of government spending. Note that the time path of spending is what is relevant, not the time path of taxes, because Ricardian equivalence holds in this model.

The steady state of the economy with government spending is very similar to the steady state described previously. In particular the relation between the growth rate and the interest rate is the same, and the output–capital ratio is the same. The ratio of private plus government consumption to output is also unchanged, which means that the private consumption–output ratio is reduced by the government spending–output ratio.

The addition of government spending does not have an important effect on the economy's response to technology shocks. The only effect comes from the fact that the loglinear approximate capital accumulation equation is now

$$k_{t+1} \approx \lambda_1 k_t + \lambda_2(a_t + n_t) + \lambda_4 x_t + (1 - \lambda_1 - \lambda_2 - \lambda_4)c_t, \tag{51}$$

where

$$\lambda_4 \equiv \frac{-(r + \delta)X/Y}{(1 - \alpha)(1 + g)}. \tag{52}$$

If the steady-state government spending–output ratio is 0.2, then $\lambda_4 = 0.02$ at the benchmark values of the other parameters. The effect of log consumption on log capital is therefore reduced by 0.02. The previous analysis of technology shocks applies if one replaces $(1 - \lambda_1 - \lambda_2)$ by $(1 - \lambda_1 - \lambda_2 - \lambda_4)$throughout.

Similar reasoning shows that the technology shock process does not affect the economy's response to government spending shocks. For simplicity, I shall therefore ignore technology shocks in the remainder of this section. Assuming an AR(1) process for government spending, the loglinear model with separable utility over consumption and leisure becomes (51) with a_t set to 0, together with

$$E_t \Delta c_{t+1} = \lambda_3 E_t(n_{t+1} - k_{t+1}), \tag{53}$$

$$x_t = \phi x_{t-1} + \varepsilon_t, \tag{54}$$

$$n_t = v[(1 - \alpha)k_t - c_t], \tag{55}$$

where $v = v(\sigma_n)$ is as defined in eq. (42).

Table 6

Consumption and capital elasticities for the separable variable-labor model with government spending shocks and lump-sum taxation.[a]

	σ_n				
ϕ	0	0.2	1	5	∞
0.00	0.70, -0.02	0.66, -0.02	0.60, -0.01	0.55, -0.01	0.53, -0.01
	0.96, -0.02	0.96, -0.02	0.95, -0.02	0.93, -0.02	0.93, -0.02
0.50	0.70, -0.03	0.66, -0.03	0.60, -0.03	0.55, -0.02	0.53, -0.02
	0.96, -0.02	0.96, -0.02	0.95, -0.02	0.93, -0.02	0.93, -0.02
0.95	0.70, -0.18	0.66, -0.16	0.60, -0.12	0.55, -0.10	0.53, -0.09
	0.96, -0.01	0.96, -0.01	0.95, -0.00	0.93, 0.00	0.93, 0.00
1.00	0.70, -0.36	0.66, -0.30	0.60, -0.21	0.55, -0.16	0.53, -0.14
	0.96, 0.00	0.96, 0.00	0.95, 0.01	0.93, 0.02	0.93, 0.02

[a] σ_n is the elasticity of labor supply and ϕ is the persistence of the AR(1) government spending shock. The model is specified in eqs. (50)–(55) in the text. The top two numbers in each group are η_{ck}, η_{cx}, where η_{ck} is the elasticity of consumption with respect to the capital stock and η_{cx} is the elasticity of consumption with respect to government spending. The bottom two numbers in each group are η_{kk}, η_{kx}, where η_{kk} is the elasticity of next period's capital stock with respect to this period's capital stock and η_{kx} is the elasticity of next period's capital stock with respect to this period's government spending.

This model can be solved in the standard fashion. (Details are given in appendix B.) Once the elasticities of consumption η_{ck} and η_{cx} have been found, the other elasticities follow straightforwardly from (51), (55), and the production function. Table 6 gives the consumption and capital elasticities, and table 7 gives the employment and output elasticities for the standard range of parameter values.

Table 6 shows that private consumption falls when government spending increases. It falls by more when government spending is more persistent, for permanent income reasons. It falls by less when labor supply is more elastic, for then increased labor supply (shown in table 7) can meet some of the increased tax burden. Labor supply increases with government spending, since the real wage is unchanged by a government spending shock and the marginal utility of consumption increases. Labor supply increases by more when labor supply is more elastic and when a more persistent change in government spending leads to a greater decline in consumption and increase in the marginal utility of consumption.

It follows from this that the output effect of government spending increases with the persistence of government spending. This is directly contrary to the claims of Barro (1981) and Hall (1980). Aiyagari, Christiano, and Eichenbaum (1992) and Baxter and King (1993) have already established the correct result in

Table 7

Employment and output elasticities for the separable variable-labor model with government spending shocks and lump-sum taxation.[a]

ϕ	σ_n				
	0	0.2	1	5	∞
0.00	0.00, 0.00	− 0.11, 0.01	− 0.31, 0.02	− 0.51, 0.03	− 0.60, 0.04
	0.33, 0.00	0.26, 0.00	0.12, 0.01	− 0.01, 0.02	− 0.07, 0.02
0.50	0.00, 0.00	− 0.11, 0.01	− 0.31, 0.03	− 0.51, 0.05	− 0.60, 0.06
	0.33, 0.00	0.26, 0.01	0.12, 0.02	− 0.01, 0.04	− 0.07, 0.04
0.95	0.00, 0.00	− 0.11, 0.05	− 0.31, 0.15	− 0.51, 0.23	− 0.60, 0.27
	0.33, 0.00	0.26, 0.04	0.12, 0.10	− 0.01, 0.16	− 0.07, 0.18
1.00	0.00, 0.00	− 0.11, 0.11	− 0.31, 0.26	− 0.51, 0.38	− 0.60, 0.43
	0.33, 0.00	0.26, 0.07	0.12, 0.17	− 0.01, 0.25	− 0.07, 0.29

[a]σ_n is the elasticity of labor supply and ϕ is the persistence of the AR(1) government spending shock. The model is specified in eqs. (50)–(55) in the text. The top two numbers in each group are η_{nk}, η_{nx}, where η_{nk} is the elasticity of employment with respect to the capital stock and η_{nx} is the elasticity of employment with respect to government spending. The bottom two numbers in each group are η_{yk}, η_{yx}, where η_{yk} is the elasticity of output with respect to the capital stock and η_{yx} is the elasticity of output with respect to government spending.
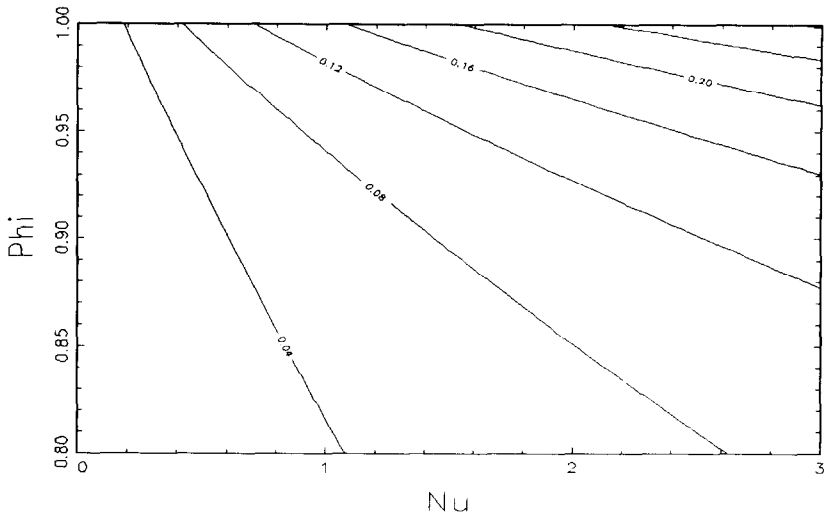
Fig. 7. Initial output response to a government spending shock with variable labor supply, separable utility, and lump-sum taxation.

The contours show the short-run elasticity of output with respect to government spending in a model with variable labor supply, additively separable utility over consumption and leisure, and lump-sum taxation. The model is specified in eqs. (50)–(55) in the text. The elasticity is plotted for different values of the parameters v and ϕ, where v is a function of the elasticity of labor supply defined in eq. (42) and ϕ is the persistence of government consumption shocks. The contour lines are 0.04 apart. Note that the smallest value of ϕ shown is 0.8, and that when $v = 0$, the elasticity is 0 for any value of ϕ.

a real business cycle framework, but the analytical approach here may make the result more transparent. Fig. 7 is a contour plot of the output elasticity against the persistence ϕ of government spending and the parameter v measuring the elasticity of labor supply. As ϕ and v approach their maximum possible values, the output elasticity approaches its maximum of 0.29. Dividing by the steady-state ratio of government spending to output (assumed to be 0.2), this implies that an extra dollar of government spending generates at most 1.45 dollars of output. The elasticity declines very rapidly with ϕ; even when $\phi = 0.95$ the largest possible elasticity is only 0.18, implying that an extra dollar of government spending generates less than an extra dollar of output.

4.2. Distortionary taxation

Distortionary taxation can be modelled in a simple way by assuming that tax is levied at a flat rate τ_t on all gross output [Baxter and King (1993)]. Once taxation is distortionary, the timing of taxation can have real effects even in

a model with an infinitely-lived representative agent; for simplicity I assume here that the government budget is balanced each period, so that

$$\tau_t = X_t / Y_t. \tag{56}$$

As in the discussion of lump-sum taxation, I assume that technology is nonstochastic and normalize it to unity. I write after-tax output as Y_t^*, defined by

$$Y_t^* \equiv (1 - \tau_t) Y_t = (1 - \tau_t) N_t^\alpha K_t^{1-\alpha}. \tag{57}$$

Then the capital accumulation equation can be written as

$$K_{t+1} = (1 - \delta) K_t + Y_t^* - C_t - X_t. \tag{58}$$

The first-order condition for optimal consumption choice, eq. (5), continues to hold but the rate of return on capital must be measured after tax as

$$R_{t+1} \equiv (1 - \alpha)(1 - \tau_{t+1}) \left(\frac{N_{t+1}}{K_{t+1}} \right)^\alpha + (1 - \delta). \tag{59}$$

The first-order condition for optimal labor supply, eq. (36), becomes

$$\theta(1 - N_t)^{-\gamma_n} = (1 - \tau_t) \frac{W_t}{C_t} = \alpha \frac{(1 - \tau_t)}{C_t} \left(\frac{K_t}{N_t} \right)^{1-\alpha}. \tag{60}$$

Comparison of eqs. (57) to (60) with eqs. (1), (2), (35), and (36) shows that a model of after-tax output Y_t^* with gross output taxation and a balanced government budget takes exactly the same form as a model of pre-tax output Y_t with technology shocks.[10] Y_t^* and $(1 - \tau_t)$ appear everywhere that Y_t and A_t^α appeared in the technology shock model. Hence the results of section 3 can be used to calculate the effects of distortionary tax shocks on after-tax output.

In doing this calculation, several points require careful attention. First, section 3 reported the effects of a 1% positive shock to technology, which corresponds to an $\alpha\%$ positive shock to $(1 - \tau_t)$. Linearizing around a steady-state value of 0.8 for $1 - \tau_t$, this corresponds to a reduction in the gross output tax rate of $0.8\alpha = 0.53$ percentage points. Second, the elasticities reported in table 3 for pre-tax output apply here to after-tax output. Noting that from eq. (57) $y_t = y_t^* - \log(1 - \tau_t)$, to get elasticities for pre-tax output one must

Table 8

Consumption and capital elasticities for the separable variable-labor model with government spending shocks and distortionary gross output taxation.[a]

ϕ	σ_n				
	0	0.2	1	5	∞
0.00	0.62, -0.02	0.60, -0.02	0.56, -0.03	0.52, -0.05	0.50, -0.07
	0.96, -0.03	0.96, -0.04	0.95, -0.06	0.94, -0.10	0.93, -0.13
0.50	0.62, -0.03	0.60, -0.03	0.56, -0.05	0.52, -0.07	0.50, -0.09
	0.96, -0.03	0.96, -0.04	0.95, -0.06	0.94, -0.09	0.93, -0.12
0.95	0.62, -0.09	0.60, -0.10	0.56, -0.13	0.52, -0.16	0.50, -0.17
	0.96, -0.02	0.96, -0.03	0.95, -0.04	0.94, -0.05	0.93, -0.06
1.00	0.62, -0.16	0.60, -0.17	0.56, -0.19	0.52, -0.20	0.50, -0.22
	0.96, -0.02	0.96, -0.02	0.95, -0.02	0.94, -0.03	0.93, -0.03

[a]σ_n is the elasticity of labor supply and ϕ is the persistence of the AR(1) government spending shock. The model is specified in eqs. (56)–(60) in the text. The top two numbers in each group are η_{ck}, η_{cx}, where η_{ck} is the elasticity of consumption with respect to the capital stock and η_{cx} is the elasticity of consumption with respect to government spending. The bottom two numbers in each group are η_{kk}, η_{kx}, where η_{kk} is the elasticity of next period's capital stock with respect to this period's capital stock and η_{kx} is the elasticity of next period's capital stock with respect to this period's government spending.

subtract α from the η_{ya} values reported in table 3. When one does this, one finds that tax cuts have small positive effects on pre-tax output whenever the elasticity of labor supply is positive. These effects increase with the elasticity of labor supply and are larger when tax cuts are temporary. Third, the analysis of technology shocks assumed that a_t followed an AR(1) process. This is equivalent here to assuming that $\log(1 - \tau_t)$ follows an AR(1) process. But if one loglinearizes eq. (56), one finds $x_t \approx y_t - 4\log(1 - \tau_t)$. Since y_t is an ARMA(2,1), an AR(1) for $\log(1 - \tau_t)$ generally implies a more complicated ARMA(2,1) process for government spending x_t.[11]

In order to allow a more direct comparison between the effects of AR(1) government spending shocks with lump-sum taxation and the effects of the same spending shocks with distortionary taxation, one can assume that x_t follows an AR(1) process and analyze the model (56) through (60) directly. Loglinearizing the model and applying the method of undetermined coefficients in the usual way (the details are given in appendix B), I obtain results reported in tables 8 and 9. Comparing these results with those in tables 6 and 7, it is clear that the

[11]Since x_t equals tax revenue, the loglinearization of (56) also implies that tax cuts always lower tax revenue because for all parameter values considered $\eta_{ya} - 4\alpha < 0$. This result follows from the fact that the benchmark steady-state tax rate of 0.2 is on the upward-sloping portion of the Laffer Curve. Temporary tax cuts reduce revenue less than permanent tax cuts, however.

Table 9

Employment and output elasticities for the separable variable-labor model with government spending shocks and distortionary gross output taxation.[a]

ϕ	σ_n				
	0	0.2	1	5	∞
0.00	0.00, 0.00	-0.07, -0.09	-0.21, -0.32	-0.38, -0.73	-0.48, -1.08
	0.33, 0.00	0.29, -0.06	0.19, -0.22	0.08, -0.49	0.01, -0.71
0.50	0.00, 0.00	-0.07, -0.08	-0.21, -0.30	-0.38, -0.68	-0.48, -0.98
	0.33, 0.00	0.29, -0.05	0.19, -0.20	0.08, -0.45	0.01, -0.66
0.95	0.00, 0.00	-0.07, -0.06	-0.21, -0.18	-0.38, -0.35	-0.48, -0.47
	0.33, 0.00	0.29, -0.04	0.19, -0.12	0.08, -0.24	0.01, -0.31
1.00	0.00, 0.00	-0.07, -0.03	-0.21, -0.09	-0.38, -0.16	-0.48, -0.21
	0.33, 0.00	0.29, -0.02	0.19, -0.06	0.08, -0.11	0.01, -0.14

[a] σ_n is the elasticity of labor supply and ϕ is the persistence of the AR(1) government spending shock. The model is specified in eqs. (56)–(60) in the text. The top two numbers in each group are η_{nk}, η_{nx}, where η_{nk} is the elasticity of employment with respect to the capital stock and η_{nx} is the elasticity of employment with respect to government spending. The bottom two numbers in each group are η_{yk}, η_{yx}, where η_{yk} is the elasticity of output with respect to the capital stock and η_{yx} is the elasticity of output with respect to government spending.

negative incentive effects of higher taxes outweigh the positive effects of higher government spending on pre-tax output. Output always falls when government spending increases; it falls by more when the elasticity of labor supply is high, and when the spending increase is temporary. A temporary spending increase leads to intertemporal substitution of work effort away from the period in which output taxation is high. Consumption also falls when government spending increases, but for permanent income reasons it falls by more when the spending increase is permanent.

5. Conclusion

In this paper I have argued that an analytical approach to the stochastic growth model helps to generate important insights. I have assumed plausible benchmark values for model parameters describing the steady-state growth path of the economy, and have used an approximate analytical solution to explore the effects of other parameters – the intertemporal elasticity of substitution in consumption, the elasticity of labor supply, the persistence of technology shocks, and the persistence of government spending shocks – on the dynamic behavior of the model. Some of the main results of this exploration are as follows.

First, a model with fixed labor supply and a very small intertemporal elasticity of substitution in consumption is a general equilibrium version of the

permanent income theory of consumption. It has many of the properties discussed informally by Fama (1992); in particular, temporary technology shocks cause temporary fluctuations in output and investment, but not in consumption. This model is also consistent with the empirical evidence of Barro and Sala-i-Martin (1992) that output converges only very slowly to its steady-state growth path.

Second, with variable labor supply it is possible for the elasticity of output with respect to technology shocks to exceed 1. This seems to be important if output fluctuations are to be explained by technology shocks, because it permits output to decline when technology grows more slowly than normal; with a smaller-than-unit elasticity, on the other hand, technology declines are needed to produce output declines. Unfortunately, an elasticity greater than 1 depends both on highly elastic labor supply (as is well understood) and on low persistence of technology shocks. If technology is a random walk and utility is separable over consumption and leisure, then even with infinitely elastic labor supply the output elasticity cannot exceed 1.

Third, the basic analysis in this paper assumes an AR(1) log technology shock. However, different solutions can be combined to obtain the solution for any linear combination of AR(1) processes. This enables me to calculate the response of the economy to a highly persistent technology shock of the type that may have occurred in the 'productivity slowdown' of the 1970's. The output elasticity with respect to such a shock can actually be negative, because low technology growth today signals even lower technology (relative to trend) in the future, and this stimulates output today rather than dampening it.

Fourth, all the models examined have the feature that expected and realized returns on capital are extremely stable. A 1% technology shock moves the realized return on a one-period investment in capital by no more than 12 basis points (at an annual rate) in a fixed-labor model and by no more than 32 basis points in a separable variable-labor model; and for most parameter values the return on capital is much less responsive to technology shocks. The reason for this stability is that most of the return on a one-period capital investment is undepreciated capital rather than the output which is affected by technology. This feature of the stochastic growth model makes it hard for the model to explain the observed variability of real interest rates.

Fifth, the paper follows recent work showing that permanent shocks to unproductive government spending, financed by lump-sum taxation, have larger output effects than temporary shocks. With sufficiently elastic labor supply and sufficiently persistent government spending, it is possible for a dollar of government spending to stimulate more than a dollar of additional output; however this requires an AR(1) process for government spending with a persistence parameter above 0.96.

Finally, it is shown that positive shocks to unproductive government spending, financed by contemporaneous distortionary taxation, are closely analogous

to negative technology shocks. An increase in spending financed by a flat-rate gross output tax reduces output; this effect is stronger when the spending increase is temporary.

The analytical approach of this paper can be used to study a number of other interesting issues. It should be straightforward, for example, to allow for convex adjustment costs in investment [Baxter and Crucini (1993)]; this might help the model to generate more variable returns to capital. The introduction of a non-market or home production technology [Benhabib, Rogerson, and Wright (1991), Greenwood and Hercowitz (1991)] might relax the restrictions on utility implied by balanced growth when labor supply is variable. There is much work to be done on more general models of distortionary taxation, including of course models in which government debt breaks the link between the time path of spending and the time path of tax rates [Greenwood and Huffman (1991), McGrattan (1993), Ohanian (1993)]. Alternative models of consumer behavior can be explored, including models of habit formation [Abel (1990), Constantin-ides (1989)] and 'rule-of-thumb' behavior [Campbell and Mankiw (1989)]. There are also interesting alternative models of production, in particular those with external effects of investment on technology [Baxter and King (1990)]. More challenging will be to allow for real and nominal macroeconomic rigidities of the type emphasized by recent work in the Keynesian tradition. Ultimately, a stochastic growth model incorporating such rigidities holds out the promise of a new synthesis in macroeconomics.

Appendix A: Taylor approximations

To obtain eq. (13) I proceed as follows. On the left-hand side of (12) is the nonlinear function $f_1(\Delta k_{t+1}) \equiv \log[\exp(\Delta k_{t+1}) - (1 - \delta)]$. This is approximated as $f_1(\Delta k_{t+1}) \approx f_1(g) + f'_1(g)(\Delta k_{t+1} - g)$, where

$$f'_1(g) = \frac{\exp(g)}{\exp(g) - (1 - \delta)} \approx \frac{1 + g}{\delta + g}. \tag{A.1}$$

On the right-hand side of (12) is the nonlinear function $f_2(c_{t+1} - y_{t+1}) \equiv \log[1 - \exp(c_t - y_t)]$.[12] This is approximated as $f_2(c_{t+1} - y_{t+1}) \approx f_2(c - y) + f'_2(c - y)(c_{t+1} - y_{t+1} - (c - y))$, where

$$f'_2(c - y) \approx 1 - \frac{r + \delta}{(1 - \alpha)(g + \delta)}. \tag{A.2}$$

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[12]Connoisseurs will recognize this as the function approximated in Campbell and Shiller (1988) and Campbell (1993).

Substituting these approximations into (12) and dropping constants, I obtain a loglinear approximate accumulation equation,

$$\left(\frac{1+g}{\delta+g}\right)\Delta k_{t+1} \approx y_t - k_t + \left[1 - \frac{r+\delta}{(1-\alpha)(g+\delta)}\right](c_t - y_t). \tag{A.3}$$

The log production function (11) can be used to substitute out y_t from this equation, yielding (13).

To obtain eq. (16), I take logs of (4) to obtain

$$r_{t+1} = f_3(a_{t+1} - k_{t+1})$$

$$\equiv \log[1 - \delta + (1-\alpha)\exp(\alpha a_{t+1} - \alpha k_{t+1})]. \tag{A.4}$$

The function $f_3(a_{t+1} - k_{t+1})$ is approximated as $f_3(a_{t+1} - k_{t+1}) \approx f_3(a - k) + f'_3(a-k)(a_{t+1} - k_{t+1} - (a-k))$, where

$$f'_3(a-k) \approx \frac{\alpha(r+\delta)}{1+r}. \tag{A.5}$$

Substituting these expressions into (15) yields (16).

To obtain eq. (33), I proceed as follows. Bellman's equation states that

$$V_t^{1-\gamma} = \max C_t^{1-\gamma} + \beta E_t V_{t+1}^{1-\gamma}. \tag{A.6}$$

It is straightforward to show that in steady state

$$\frac{V_t^{1-\gamma}}{C_t^{1-\gamma}} = \frac{r-g}{1+r}. \tag{A.7}$$

Taking logs of (A.6) and dropping constants, I obtain $f_2((1-\gamma)(c_t - v_t)) \equiv \log(1 - \exp[(1-\gamma)(c_t - v_t)]) = (1-\gamma)E_t(v_{t+1} - v_t)$. On the left-hand side is the same function approximated above in (A.2), where now

$$f'_2((1-\gamma)(c-v)) \approx -\frac{r-g}{1+g} = 1 - \lambda_1. \tag{A.8}$$

To obtain eq. (40), I take logs of (36), dropping constants, and obtain

$$-\gamma_n \log[1 - \exp(n_t)] = \alpha a_t + (1-\alpha)(k_t - n_t) - c_t. \tag{A.9}$$

On the left-hand side of (A.9) is the nonlinear function $f_4(n_t) \equiv \log[1 - \exp(n_t)]$. This is approximated as $f_4(n_t) \approx f_4(n) + f'_4(n)(n_t - n)$, where

$$f'_4(n) = -\frac{\exp(n)}{1 - \exp(n)} \approx -\frac{N}{1 - N}. \tag{A.10}$$

Substituting into (A.9) and rearranging, I obtain (40) and (41).

Appendix B: Complete solutions for the variable-labor models

In the separable variable-labor model with technology shocks, the solution for η_{ck} is the solution to the quadratic eq. (24), $Q_2\eta_{ck}^2 + Q_1\eta_{ck} + Q_0 = 0$, where now

$$Q_2 \equiv [1 + \lambda_3 v][1 - \lambda_1 - \lambda_2(1 + v)],$$

$$Q_1 \equiv [1 + \lambda_3 v][\lambda_1 + \lambda_2(1 - \alpha)v]$$

$$- \lambda_3[(1 - \alpha)v - 1][1 - \lambda_1 - \lambda_2(1 + v)] - 1,$$

$$Q_0 \equiv -\lambda_3[(1 - \alpha)v - 1][\lambda_1 + \lambda_2(1 - \alpha)v]. \tag{B.1}$$

The solution is given by the quadratic formula (26), as before. Given η_{ck}, the solution for η_{ca} follows as

$$\eta_{ca} = \frac{(1 + \alpha v)[\lambda_3\phi - \lambda_2\{\eta_{ck}(1 + \lambda_3 v) - \lambda_3((1 - \alpha)v - 1)\}]}{\{\eta_{ck}(1 + \lambda_3 v) - \lambda_3((1 - \alpha)v - 1)\}\{1 - \lambda_1 - \lambda_2(1 + v)\} - \{1 - \phi(1 + \lambda_3 v)\}}. \tag{B.2}$$

In the nonseparable variable-labor model with technology shocks, the solution for consumption elasticities is more complicated. It is easiest to state by defining some intermediate parameters:

$$\psi \equiv (1 - \rho)(1 - \gamma)(N/(1 - N))v(1),$$

$$\xi_1 \equiv 1 - \rho(1 - \gamma) - \psi + \lambda_3 v(1),$$

$$\xi_2 \equiv -(1 - \rho(1 - \gamma) - \psi),$$

$$\xi_3 \equiv \psi(1 - \alpha) - \lambda_3[v(1)(1 - \alpha) - 1],$$

$$\xi_4 \equiv -\psi(1-\alpha),$$

$$\xi_5 = \psi\alpha - \lambda_3[1 + v(1)\alpha],$$

$$\xi_6 \equiv -\psi\alpha,$$

$$\kappa_1 \equiv \lambda_1 + \lambda_2(1-\alpha)v(1),$$

$$\kappa_2 \equiv 1 - \lambda_1 - \lambda_2(1 + v(1)),.$$

$$\kappa_3 \equiv \lambda_2(1 + \alpha v(1)). \tag{B.3}$$

Then again η_{ck} solves $Q_2\eta_{ck}^2 + Q_1\eta_{ck} + Q_0 = 0$, where now

$$Q_2 \equiv \xi_1\kappa_2, \quad Q_1 \equiv \xi_1\kappa_1 + \xi_3\kappa_2 + \xi_2, \quad Q_0 \equiv \xi_3\kappa_1 + \xi_4. \tag{B.4}$$

The solution is given by the quadratic formula (26), as before. Given η_{ck}, the solution for η_{ca} follows as

$$\eta_{ca} = -\frac{\phi\xi_5 + \xi_6 + \kappa_3[\xi_1\eta_{ck} + \xi_3]}{\phi\xi_1 + \xi_2 + \kappa_2[\xi_1\eta_{ck} + \xi_3]}. \tag{B.5}$$

In the separable variable-labor model with government spending shocks and lump-sum taxation, the quadratic equation for η_{ck} has the same parameters as before, except that $1 - \lambda_1 - \lambda_2(1 + v)$ is replaced by $1 - \lambda_1 - \lambda_2(1 + v) - \lambda_4$ everywhere. The solution for η_{cx} follows as

$$\eta_{cx} = \frac{\lambda_4[\lambda_3((1-\alpha)v - 1 - v\eta_{ck}) - \eta_{ck}]}{\phi(1 + \lambda_3 v) - 1 - [\lambda_3((1-\alpha)v - 1 - v\eta_{ck}) - \eta_{ck}](1 - \lambda_1 - \lambda_2(1 + v) - \lambda_4)}. \tag{B.6}$$

In the separable variable-labor model with government spending shocks and distortionary output taxation, the loglinear capital accumulation equation becomes

$$k_{t+1} \approx \lambda_1^* k_t + \lambda_2^* n_t + \lambda_4^* x_t + (1 - \lambda_1^* - \lambda_2^* - \lambda_4^*)c_t, \tag{B.7}$$

where $\lambda_1^* = \lambda_1 - (\tau/(1-\tau))\lambda_2 - 1/(1-\tau)\lambda_4$, $\lambda_2^* = \lambda_2/(1-\tau)$, and $\lambda_4^* = \lambda_4/(1-\tau)$.

The solution is once again easiest to state by defining some intermediate parameters:

$$v^* \equiv (1 - N)\sigma_n/(N + (1 - N)\sigma_n[1 - \alpha/(1 - \tau)]),$$

$$v_1 \equiv (1 - \alpha)v^*/(1 - \tau),$$

$$v_2 \equiv -\tau v^*/(1 - \tau),$$

$$v_3 \equiv -v^*,$$

$$\psi_1 \equiv \tau(1 - \alpha)/\alpha(1 - \tau) - 1,$$

$$\psi_2 \equiv 1/(1 - \tau),$$

$$\psi_3 \equiv -\tau/\alpha(1 - \tau),$$

$$\lambda_1^{**} \equiv \lambda_1^* + \lambda_2^* v_1,$$

$$\lambda_2^{**} \equiv \lambda_4^* + \lambda_2^* v_2,$$

$$\lambda_3^{**} \equiv 1 - \lambda_1^* - \lambda_2^*(1 - v_3) - \lambda_4^*,$$

$$\zeta_1 \equiv \lambda_3(\psi_1 + \psi_2 v_1),$$

$$\zeta_2 \equiv \lambda_3(\psi_3 + \psi_2 v_2),$$

$$\zeta_3 \equiv \lambda_3\psi_2 v_3. \tag{B.8}$$

Once again η_{ck} solves $Q_2\eta_{ck}^2 + Q_1\eta_{ck} + Q_0 = 0$, where now

$$Q_2 \equiv (1 - \zeta_3)\lambda_3^{**}, \quad Q_1 \equiv (1 - \zeta_3)\lambda_1^{**} - \zeta_1\lambda_3^{**} - 1, \quad Q_0 \equiv -\zeta_1\lambda_1^{**}.$$

$$\tag{B.9}$$

Given η_{ck}, the solution for η_{cx} follows as

$$\eta_{cx} = \frac{\zeta_2\phi + [\zeta_1 - (1 - \zeta_3)\eta_{ck}]\lambda_2^{**}}{(1 - \zeta_3)\phi - [\zeta_1 - (1 - \zeta_3)\eta_{ck}]\lambda_3^{**} - 1}. \tag{B.10}$$

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