Supplementary Notes on Chapter 5 of D. Romer's Advanced Macroeconomics Textbook (4th Edition)

Changsheng Xu & Ming Yi

School of Economics, Huazhong University of Science and Technology

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Interpreting ρ_A and ρ_G

$$\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \varepsilon_{A,t},\ -1 < \rho_A < 1 \tag{5.9}$$

$$\tilde{G}_t = \rho_G \tilde{G}_{t-1} + \varepsilon_{G,t},\ -1 < \rho_G < 1 \tag{5.11}$$

- Merits of Stationary processes? Effects of a sudden shock keep diminishing. Extreme case: $\rho = 0$.
- $E(\tilde{A}_t) = E(\tilde{G}_t) = 0$.
- Q: Considering the Endogenous Growth ingredients, $\rho_A \lesseqqgtr 0$?
- Q: Considering the Government spending rules, $\rho_G \lesseqqgtr 0$?

Interpreting Equation (5.24)

$$(5.23) \qquad \frac{1}{c_t} = e^{-\rho} E_t \left[\frac{1}{c_{t+1}} (1 + r_{t+1}) \right]$$

$$(5.24) \qquad \frac{1}{c_t} = e^{-\rho} \left[E_t \left(\frac{1}{c_{t+1}} \right) E_t(1 + r_{t+1}) + Cov \left(\frac{1}{c_{t+1}}, 1 + r_{t+1} \right) \right]$$

- Euler Equation with uncertainty.
- $Cov(X, Y) = E[(X - E(X))(Y - E(Y))]$. Interpreted as how the patterns of the realization bundles of c_{t+1} and r_{t+1} (they are both random variables) would look like.
- If $Cov < 0$, households know that it is more likely (compared to the baseline case where $Cov = 0$) that if a higher next-period interest rate occurs, a lower next-period marginal utility (on consumption) also occurs. As a result, given other conditions fixed, i.e, given $E_t(\frac{1}{c_{t+1}})$ and $E_t(1 + r_{t+1})$, the households want to save less (higher c_t).

Interpreting Equation (5.31)

$$\text{(5.23)} \qquad \frac{1}{c_t} = e^{-\rho} E_t \left[\frac{1}{c_{t+1}} (1 + r_{t+1}) \right]$$

$$\text{(5.31)} \qquad \ln s_t - \ln(1 - s_t) = -\rho + n + \ln \alpha + \ln E_t \left(\frac{1}{1 - s_{t+1}} \right)$$

- If households choose $\hat{s} = \ln \alpha + n - \rho$ in each period, then there is no uncertainty in s_{t+1} and $\hat{s} = s_t = s_{t+1}$ solves (5.31).
- Economic intuition? In this specific and oversimplified model (with $\delta = 0$ and $G_t \equiv 0$), the technological shock, say a positive one, leads to both a decrease in $\frac{1}{c_{t+1}}$ (given other things fixed) and an increase in r_{t+1} (why?). Magically, in this model, the two effects offset each other:

Interpreting Equation (5.31) (Continued)

$$\frac{1+r_{t+1}}{c_{t+1}} = \frac{\alpha\left(\frac{A_{t+1}L_{t+1}}{K_{t+1}}\right)^{1-\alpha}}{(1-s_{t+1})\,Y_{t+1}\cdot\frac{1}{N_{t+1}}} = \frac{\alpha\,Y_{t+1}K_{t+1}^{-1}}{(1-s_{t+1})\,Y_{t+1}}.$$

- The two Y_{t+1}'s offset each other. According to (5.27), there is no uncertainty in K_{t+1} at time t. As a result, if s_{t+1} is fixed at $\hat{s}$, there is no need for the households to adjust c_t according to the contingent technological shock $\tilde{A}_{t+1}$.
- Any other equilibrium paths? There may be, but isn't an equilibrium path with $s_t \equiv \hat{s}$ beautiful and intuitive?

Interpreting Equation (5.37)

- it is thus straightforward to understand equation (5.37), given $\frac{c_t}{1-\ell_t} = \frac{w_t}{b}$ (5.26), $c_t = Y_t \frac{1-\hat{s}}{N_t}$, $w_t = Y_t \frac{1-\alpha}{\ell_t N_t}$: First, ℓ_t does not need to adjust according to $\tilde{A}_{t+1}$. Second, shock $\tilde{A}_t$ does not change ℓ_t, a current-period positive technological shock tends to increase income and to decrease marginal utility on consumption, these two effects offset each other. Consequently, we have $\ell_t \equiv \hat{\ell} = \frac{1-\alpha}{1-\alpha+b(1-\hat{s})}$.
- Again, we get fixed $\hat{\ell}$ and $\hat{s}$ only in this oversimplified model!

Interpreting Fluctuations in Y_t

$$
\begin{aligned}
(5.39)\ \ln Y_t = \quad & \alpha \ln \hat{s} + \alpha \ln Y_{t-1}^{trend} + (1-\alpha)(\bar{A} + gt + \ln \hat{\ell} + \bar{N} + nt) \\
& + \alpha \tilde{Y}_{t-1} + (1-\alpha)\tilde{A}_t \\
= \quad & \ln Y_t^{trend} + \tilde{Y}_t \ ,
\end{aligned}
$$

where

$$
\begin{aligned}
\ln Y_0^{trend} &= \alpha \ln K_0 + (1-\alpha)(\bar{A} + \ln \hat{\ell} + \bar{N}) && (1) \\
\tilde{Y}_0 &= (1-\alpha)\tilde{A}_0 = (1-\alpha)\varepsilon_{A,0} && (2) \\
(5.42) \qquad \tilde{Y}_t &= (\alpha + \rho_A)\tilde{Y}_{t-1} - \alpha\rho_A \tilde{Y}_{t-2} + (1-\alpha)\varepsilon_{A,t}
\end{aligned}
$$

- Numerical examples: $\alpha = \frac{2}{3}, \rho_A = \frac{1}{3}, \bar{A} = 1, g = 0.03, b = \frac{1}{2}, n = 0.01, \rho = 0.1, \bar{N} = 10, K_0 = 10,\ Var(\varepsilon_{A,t}) = 0.05.$

Interpreting Fluctuations in Y_t (Continued)

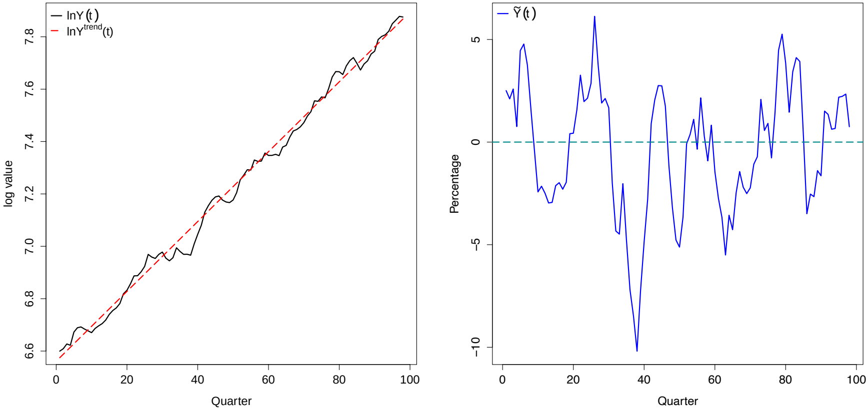


Figure 1: Numerical example with $\alpha = \frac{2}{3}$, $\rho_A = \frac{1}{3}$, $\bar{A} = 1, g = 0.03,\ b = \frac{1}{2},\ n = 0.01,\ \rho = 0.1,$ $\bar{N} = 10,\ K_0 = 10,\ Var(\varepsilon_{A,t}) = 0.05.$

The oversimplified model, Pros and Cons

Pros

- Analytically solvable.
- Delivers the idea of **Real Fluctuations** under parsimonious settings.

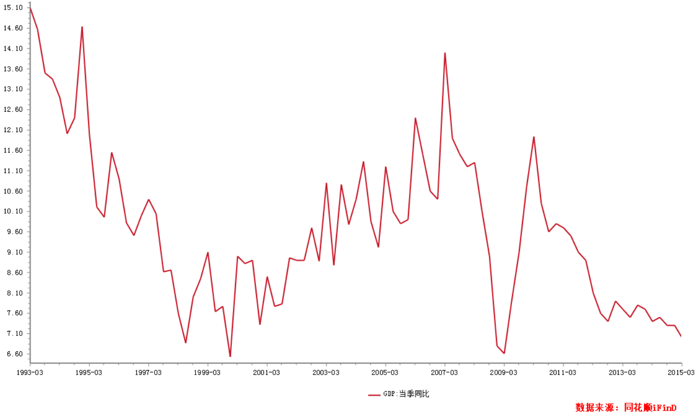
Cons

- Unrealistic predictions. Recall that $s(t) \equiv \hat{s}$, so consumptions and investments fluctuate at the same rate as the output does. Table 5.2...

An Example

$\alpha = \frac{2}{3}, g = 0.02, n = 0.0025, \delta = 0.025, \rho_A = 0.95, \rho_G = 0.95, r^* = 0.015, \ell^* = \frac{1}{3}$.
$a_{LA} = 0.35$, $a_{LK} = -0.31$, $a_{CA} = 0.38$, $a_{CK} = 0.59$, $b_{KA} = 0.08$, $b_{KK} = 0.95$, $a_{CG} = -0.13$, $a_{LG} = 0.15$, $b_{KG} = -0.004$.
For China, set $b_{KG} = 0.4 * 0.95 + 0.6 * (-0.004)$. (Why?)

$$\begin{aligned}
\tilde{Y}_t &= \ln Y_t - \ln Y_t^{trend} \\
&= \ln Y_t - \ln Y_{t-1} + \ln Y_{t-1} - \ln Y^* \\
&= \ln Y_t - \ln Y_{t-1} + \ln Y_{t-1}^* + \tilde{Y}_{t-1} - \ln Y^* \\
&= \ln \frac{Y_t}{Y_{t-1}} - \ln \frac{Y_t^*}{Y_{t-1}^*} + \tilde{Y}_{t-1} \\
&\simeq g_Y(t) - g_Y^{fundamental}(t) + \tilde{Y}_{t-1} \qquad (3)
\end{aligned}$$


15.10
14.60
14.10
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11.60
11.10
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9.60
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1993-03
1995-03
1997-03
1999-03
2001-03
2003-03
2005-03
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2009-03
2011-03
2013-03
2015-03
GDP:当季同比
数据来源：同花顺iFinD

Figure 2

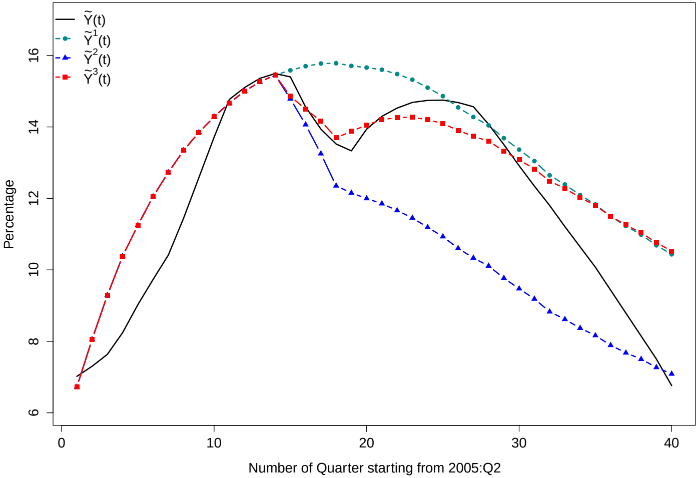

Figure 3: Period: 2nd quarter of 2005 - 1st quarter of 2015. $\tilde{Y}$ is the real outcome, $\tilde{Y}^1$ neglects the 2008 financial crisis and 2009 bailout plans, while $\tilde{Y}^2$ considers the former and $\tilde{Y}^3$ further considers the latter.

This is, of course, an informal and incorrect study because I am fitting data by treating randomly generated paths of stochastic processes, rather than observed ones, as the underlying realized shocks, besides the problem that all parameters are not calibrated to China economy. The codes and figure, however, can serve as a simple illustrating example.

Hodrick-Prescott Filter

Let $\{y_t\}_{t=1}^T$ be the logarithms of a time series variable, which is maded up of a trend component $\{y_t^*\}_{t=1}^T$ and a deviating component $\{\tilde{y}_t\}_{t=1}^T$. Given a positive value λ, there is a trend component that solves

$$\min_{\{y_t^*\}_{t=1}^T} \left(\sum_{t=1}^{T} (y_t - y^*)^2 + \lambda \sum_{t=2}^{T-1} [(y_{t+1}^* - y_t^*) - (y_t^* - y_{t-1}^*)]^2 \right) \tag{4}$$

- The first term penalizes tye deviations while the second term penalizes variations in growth rate (a trend should not be unsmooth).
- It is often suggested that $\lambda = 1600$ for quarterly data, $\lambda = \frac{1600}{4^4} = 6.25$ for annual data, and $\lambda = 1600 \times 3^4 = 129600$ for monthly data.

Revisit the numerical example

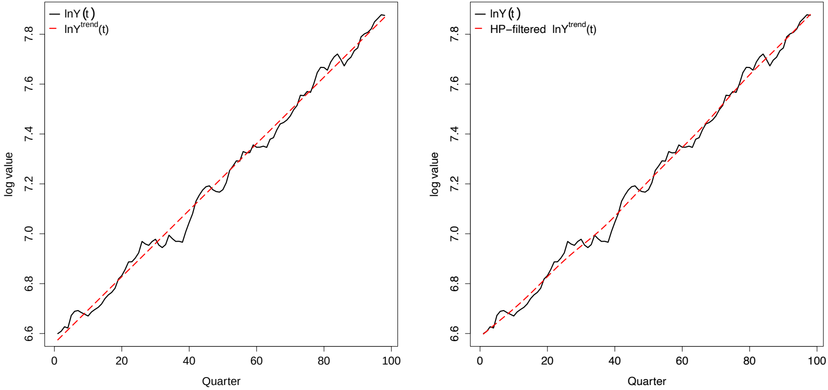

Figure 4: Real trend on left, Hodrick-Prescott filtered trend on right. Numerical example with $\alpha = \frac{2}{3}$, $\rho_A = \frac{1}{3}$, $\bar{A} = 1, g = 0.03$, $b = \frac{1}{2}$, $n = 0.01$, $\rho = 0.1$, $\bar{N} = 10$, $K_0 = 10$, $Var(\varepsilon_{A,t}) = 0.05$.

Revisit the numerical example (continued)

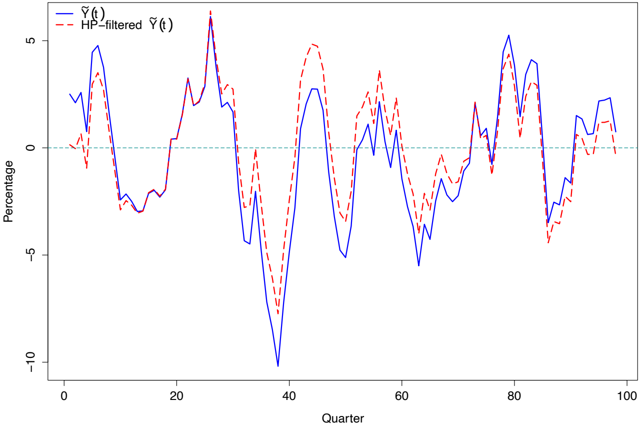

Figure 5: Real cycles versus Hodrick-Prescott filtered cycles.

Replicating Figures 5.2, 5.3, and 5.4

Practice with the codes!

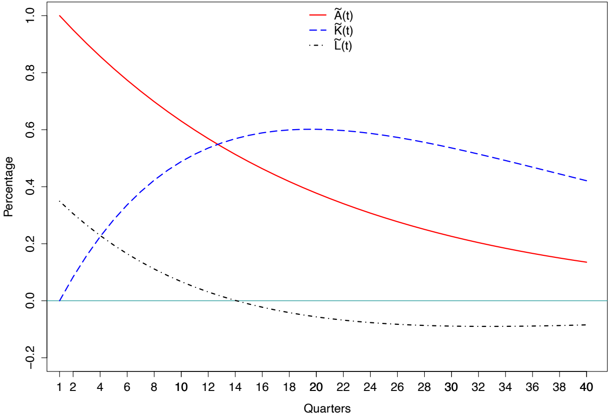

Figure 6: Replicate Figure 5.2

Replicating Figures 5.2, 5.3, and 5.4

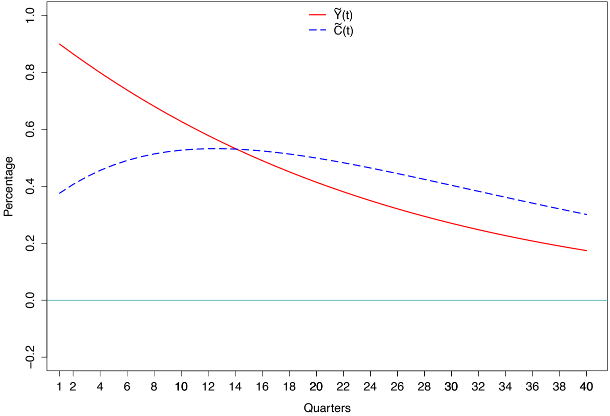

Figure 7: Replicate Figure 5.3

Replicating Figures 5.2, 5.3, and 5.4

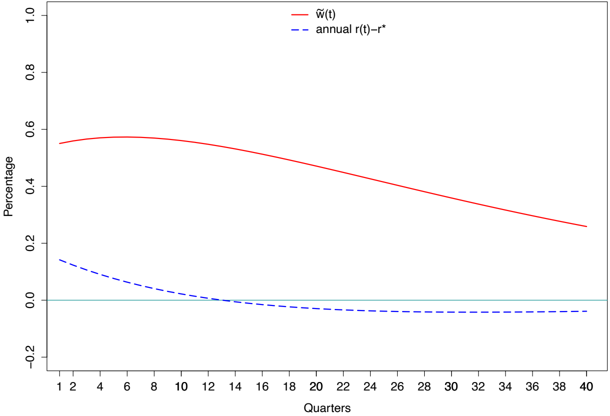

Figure 8: Replicate Figure 5.4