Advanced Macroeconomics

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Midterm Exam II (Open-Book)

Undergraduate Program in Economics, HUST

Tuesday, May/07/2019

Name: ______________________ **Student ID:**_____________________

1. ($20' \times 5 = 100$ points) Answer the questions below.

(a) Recall Equation (3.32)

$$L(i) = \left[\frac{\lambda}{p(i)}\right]^{\frac{1}{1-\phi}}. \tag{3.32}$$

Suppose a monopolist is facing a demand function depicted by equation (3.32), given that her marginal cost is fixed at c, how should the monopolist set the price of its products?

(b) Recall equation (3.37):

$$\int_{\tau=t}^{\infty} e^{-r(\tau-t)}\pi(i,\tau)\mathrm{d}\tau = \frac{w(t)}{BA(t)}. \tag{3.37}$$

What is the growth rate of term $\pi(i,\tau)$ along the equilibrium path of the model?

(c) Recall equation (3.41):

$$\pi(t) = \frac{1-\phi}{\phi}\frac{\bar{L}-L_A}{\rho+BL_A}\frac{w(t)}{A(t)}. \tag{3.41}$$

There is a typo (印刷错误) within the equation above, point it out.

(d) Recall equation (3.45)

$$\frac{\dot{Y}(t)}{Y(t)} = \max\left\{\frac{(1-\phi)^2}{\phi}B\overline{L} - (1-\phi)\rho, 0\right\}. \tag{3.45}$$

How does the equilibrium-path g change with population size $\overline{L}$? Explain the intuition behind your results.

(e) Recall equation (3.35)

$$U = \int_{t=0}^{\infty} e^{-\rho t} \ln C(t) \mathrm{d}t. \tag{3.35}$$

Now let us rewrite the objective function as

$$U = \int_{t=0}^{\infty} \frac{C(t)^{1-\theta}}{1-\theta} \mathrm{d}t, \qquad \theta > 0. \tag{3.35'}$$

With equation (3.35) replaced by (3.35′), and all other settings in you textbook remained the same, solve the Romer model (Give your revised version of equation (3.43)).

1.

(a). $TR = P(q)\cdot q$.

$MR = P(q) + q\,\dfrac{dP(q)}{dq} = P(1+\frac{1}{\varepsilon}) = P(1+\phi-1)$

$= P(\phi)$

$MC = c$

if $MR = MC$

$P(1+\frac{1}{\varepsilon}) = c$

$P(i) = \dfrac{c}{\phi}$

The monopolist will set the price of its products at $\frac{c}{\phi}$

(b).

$$\pi(t) = \frac{1-\phi}{\phi}\,\frac{\bar{L}-L_A}{A(t)}\,w(t).$$

$$\frac{\dot{\pi}(t)}{\pi(t)} = \frac{\dot{w}(t)}{w(t)} - \frac{\dot{A}(t)}{A(t)}.$$

$$= \frac{1-\phi}{\phi} B L_A - B L_A$$

$$= \frac{1-2\phi}{\phi} B L_A.$$

$$L_A = \max\{(1-\phi)\bar{L} - \frac{\phi\rho}{B}, 0\}.$$

Then $\dfrac{\dot{\pi}(t)}{\pi(t)} = \max\left\{\dfrac{1-2\phi}{\phi} B\,[(1-\phi)\bar{L} - \dfrac{\phi\rho}{B}], 0\right\}$

(C). Actually the equation should be:

$$\int_{\tau=t}^{\infty} e^{-r(\tau-t)}\,\pi(i,\tau)\,d\tau = \frac{1-\phi}{\phi}\,\frac{\bar{L}-L_A}{\rho+BL_A}\,\frac{W(t)}{A(t)}$$

(d). When population size $\bar{L}$ increases, the equilibrium-path g will also increase.

since $\dfrac{d\,\frac{\dot{Y}(t)}{Y(t)}}{d\bar{L}} > 0$. , $\dfrac{\dot{Y}(t)}{Y(t)} = \dfrac{1-\phi}{\phi}\,\dfrac{\dot{A}(t)}{A(t)} = \dfrac{1-\phi}{\phi}\,g$; $\dfrac{dg}{d\frac{\dot{Y}(t)}{Y(t)}} > 0$ then. $\dfrac{dg}{d\bar{L}} > 0$.

The intuition is that when the population size increases, more people will engage in producing technology A. Thus the growth rate of A will increase.

(e). $g_c = \frac{\dot{c}(t)}{c(t)} = \frac{r(t)-\rho}{\theta}$

Then $r(t) = \rho + \theta \frac{1-\phi}{\phi} BL_A$.

$$\int_{\tau=t}^{\infty} e^{-r(\tau-t)} \pi(i,\tau)\, d\tau = \frac{(1-\phi)(\bar{L}-L_A)}{\rho\phi + [\theta-1+(2-\theta)\phi]BL_A} \cdot \frac{W(t)}{A(t)}$$

since $\int_{\tau=t}^{\infty} e^{-r(\tau-t)} \pi(i,\tau)\, d\tau = \frac{W(t)}{BA(t)}$

Then $\frac{(1-\phi)(\bar{L}-L_A)W(t)}{\rho\phi + [\theta-1+(2-\theta)\phi]BL_A A(t)} = \frac{W(t)}{B\,A(t)}$

$$L_A = \frac{(1-\phi)B\bar{L}-\rho\phi}{B[\phi+\theta(1-\phi)]} \quad (3.43)$$

$$L_A = \max\left\{\frac{(1-\phi)B\bar{L}-\rho\phi}{B[\phi+\theta(1-\phi)]},\ 0\right\}.$$