Advanced Macroeconomics

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Midterm Exam I (Open-Book)

Undergraduate Program in Economics, HUST

Tuesday, April/02/2019

Name: ______________________ **Student ID:**_____________________

1. ($10' + 10' + 20' + 10' = 50$ points) Consider a Solow economy with a production function:

$$F\left[K(t), L(t), A(t), Z(t)\right] = \left[Z(t)K(t)\right]^{\alpha}\left[A(t)L(t)\right]^{1-\alpha}, \tag{1}$$

with $0 < \alpha < 1$, $\frac{\dot{L}(t)}{L(t)} \equiv n > 0$, $\frac{\dot{A}(t)}{A(t)} \equiv g > 0$, $Z(t)$ stands for the stock of nonrenewable resources and $\frac{\dot{Z}(t)}{Z(t)} \equiv -\gamma < 0$. Depreciation rate of capital is $\delta > 0$, saving rate is fixed at $s \in (0, 1)$.

(a) Rewrite the production function in the intensive form. Explicitly give your defition of $k(t)$ and the form of $f(\cdot)$.

(b) Given $k(t)$, what is the break-even investment now?

(c) Show that for small enough γ values (that is to say, the value of γ cannot be very large), there exists a steady state that the economy automatically converges to. You can prove it either mathematically or through illustration of figures.

(d) Given the existence of a steady state, find out the explicit expression for k^*. How does a change in the value of γ affect the steady-state k^*? Explain the economic intuition behind your answer.

2. ($20' + 20' + 10' = 50$ points) Euler's equations are prevailing in modern Macroeconomics. In a simplified infinite-horizon macroeconomic model with descrete timesteps, we often have Euler's equations in the form of:

$$U'(c_t) = \beta(1 + r_{t+1})U'(c_{t+1}), \tag{2}$$

where $U(\cdot)$ is the instantaneous utility function, $\beta \in (0, 1)$ is the utility discounting factor, and r_{t+1} stands for the real interest rate in period $t + 1$.

(a) Explain the economic intuition behind Equation (2). Do Euler's equations serve as necessary or/and sufficient conditions for the optimization problem of households?

(b) Equation (2) obviously depicts the relationship between c_t and c_{t+1}. However, if we would like to build the relationship between c_t and c_{t+2}, how should the equation be rewritten?

(c) There is a typo (印刷错误) on page 89 of your textbook (page 69 if you are using the Chinese-translated version). Point it out.

1. (a) $k(t) = \dfrac{K(t)}{Z(t)^{\frac{\alpha}{1-\alpha}} A(t) L(t)}$ 3'

$f(\cdot) = \dfrac{F(\cdot)}{Z(t)^{\frac{\alpha}{1-\alpha}} A(t) L(t)}$ 3'

$$f(k) = \frac{[Z(t)K(t)]^{\alpha}[A(t)L(t)]^{1-\alpha}}{Z(t)^{\frac{\alpha}{1-\alpha}} A(t) L(t)}$$

$$= \left[\frac{Z(t)K(t)}{Z(t)^{\frac{1}{1-\alpha}} A(t) L(t)}\right]^{\alpha}$$

$$= [k(t)]^{\alpha}$$ 4'

(b) $\dot{K}(t) = sF(\cdot) - \delta K(t)$

$$\dot{k}(t) = \frac{\dot{K}(t)[Z(t)^{\frac{\alpha}{1-\alpha}} A(t)L(t)] - K(t)[\frac{\alpha}{1-\alpha}\frac{\dot{Z}(t)}{Z(t)^{\frac{1}{1-\alpha}}} A(t)L(t) + Z(t)^{\frac{\alpha}{1-\alpha}} \dot{A}(t)L(t) + Z(t)^{\frac{\alpha}{1-\alpha}} A(t)\dot{L}(t)]}{[Z(t)^{\frac{\alpha}{1-\alpha}} A(t)L(t)]^2}$$

$$= sf(k) - \delta k(t) - k(t)\left[-\frac{\alpha}{1-\alpha}\rho + g + n\right]$$

$$= sf(k) - \left(-\frac{\alpha}{1-\alpha}\rho + n + g + \delta\right) k(t)$$ 5'

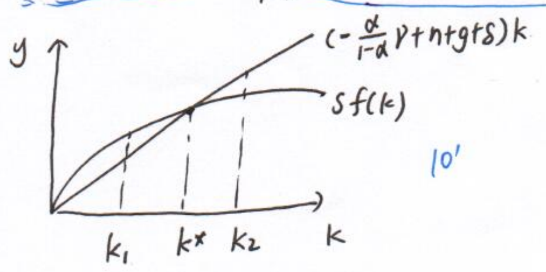
break-even investment $\left(-\frac{\alpha}{1-\alpha}\rho + n + g + \delta\right) k(t)$ 5'

(c) $f'(k) = \alpha[k(t)]^{\alpha-1} > 0$, $\lim_{k\to 0} f'(k) = \infty$ $\lim_{k\to\infty} f'(k) = 0$

$f''(k) = \alpha(\alpha-1)[k(t)]^{\alpha-2} < 0$ 10'

if $sf(k) > \left(-\frac{\alpha}{1-\alpha}\rho + n + g + \delta\right) k(t)$, $\dot{k}(t) > 0$, $k\uparrow \to k^*$

if $sf(k) < \left(-\frac{\alpha}{1-\alpha}\rho + n + g + \delta\right) k(t)$, $\dot{k}(t) < 0$, $k\downarrow \to k^*$ 10'



10'

$k_1 < k^*$时, $k\uparrow \to k^*$ 10'

$k_2 > k^*$时, $k\downarrow \to k^*$

(d) $s[k(t)]^{\alpha} = \left(-\frac{\alpha}{1-\alpha}\rho + n+g+\delta\right) k(t)$

$$k(t) = \left[\frac{s}{-\frac{\alpha}{1-\alpha}\rho + n+g+\delta}\right]^{\frac{1}{1-\alpha}}$$ 5'

$\rho\nearrow$, $k^*\nearrow$; 当 $\rho \geq \frac{(1-\alpha)(n+g+\delta)}{\alpha}$ 时, 稳态不存在 3'

资源减少的速度越快, 越需要更多的k来弥补, 这样才能使实际投资与持平投资相等, 当ρ过大时, 再多的k也不能弥补, 经济不存在稳态 2'

2. (a) t期与t+1期消费的边际效用的现值相等. 即t期节约一笔消费所带来的效用减少和将这笔消费留到t+1期所带来的效用增加的现值相等。 必要条件 20'

(b) $U'(C_t) = \beta^2(1+r_{t+1})(1+r_{t+2})U'(C_{t+2})$ 20'

并且在此后所有时期都将资本维持在K_{GR}水平 $\to k_{GR}$

(c) $nf(k) \to nk$,

$f(k_{GR} - nk_{GR})$ 大于 $f(k^*) - nk^*$ $\to f(k_{GR}) - nk_{GR}$ 大于 $f(k^*) - nk^*$ 10'