Advanced Macroeconomics

Instructed by Xu & Yi

Midterm Exam I (Open-Book)

Undergraduate Economics Program, HUST

Friday, March/31/2017

Name: ______________________ **Student ID:**______________________

1. ($10' + 20' + 10' + 10' = 50$ points) Consider an economy depicted by a Solow growth model, in which in stead of assuming a Cobb-Douglas form, we take a constant elasticity of substitution (CES) production function

$$F\left(K(t), A_L(t)L(t)\right) = \left\{\alpha\left[K(t)\right]^{\gamma} + (1-\alpha)\left[A(t)L(t)\right]^{\gamma}\right\}^{\frac{1}{\gamma}}, \tag{1}$$

where $0 < \gamma \leq 1$. All other settings of the model are kept the same as in your textbook, e.g., $L(t)$ grows at rate n, $A(t)$ grows at rate g, capital depreciates at rate δ, the government is absent, saving rate is fixed at s, etc. For simplicity, we further assume inequality $\alpha^{1/\gamma} < \frac{n+\delta+g}{s}$ always holds.

(a) Does the production function have an *intensive form*? If yes, give the expression for $f(k(t))$. If no, explain your answers.

(b) It is said that the model above admits a unique steady state. Prove it *both mathematically and graphically.*

(c) Find out the steady state k^* (k^* as a function of all the parameters).

(d) Does the steady state exhibit a balanced-growth-path feature? If yes, prove it. If no, explain why.

2. ($10' \times 5 = 50$ points) Suppose an OLG economy is evolving on this unique steady state depicted by Figure 2.11, Equations (2.60) and (2.61) in your textbook, i.e., assumptions of $\theta = 1$ and a Cobb-Douglas production function are taken. Suddenly, the government announces the introduction of a *social security system*: From now on, the government takes some wealth from the younger generation and give it to the older generation at each stage. Specifically, each young person is taxed in the amount $d > 0$, and each old person gets amount $(1 + \xi_t)d$. All other settings in your textbook, such as population grows at rate n, technology grows at rate g, and so on, still remain here.

(a) The budget constraint (2.44) in your textbook becomes incorrect here. Provide the revised version for this model.

(b) If the government makes sure $\xi_t \equiv r_t$ holds for all stages, does the introduction of the social security system make the new steady state any different from the old one?

(c) It is required that the government supports the older generation's benefits with only the taxes raised from the younger generation at each stage, and the government cannot make profits through running the social security system. Show that under these two requirements, we must have $\xi_t \equiv n$.

(d) If $\xi_t \equiv \xi = n > r$, derive the revised version of (2.60) in your textbook.

(e) The government argues that, along the current steady state, the economy is suffering from dynamic inefficiency because of over-accumulation of capital (too much k^* in the steady state). And by introducing a social security system characterized by $\xi_t \equiv \xi = n > r$, the problem of dynamic inefficiency could be (at least partly) solved. Based on your answer to (d), is the government's statement justifiable?

Solution Hints

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I did not have enough time to double-check this file while writing it. Please do me a favor by pointing out the mistakes and typos, if any, that I have made. (yiming@hust.edu.cn)

1.(a) $f(k(t)) = [\alpha k^{\gamma}(t) + (1-\alpha)]^{\frac{1}{\gamma}}$, where $k(t) = \frac{K(t)}{A(t)L(t)}$.

1.(b) Note that the specific Solow model in your textbook just takes these assumptions: CRS property on $F(\cdot)$, strict monotonicity and concavity of $f(\cdot)$, and Inada conditions on $f'(\cdot)$. It does not require a C-D form production function in your textbook!

- The CRS and strict monotonicity properties are obviously satisfied.
- With $\gamma > 0$, it is straight to have $f''(\cdot) < 0$ for all $k > 0$. The concavity requirement is guaranteed.
- Now Consider $f'(k) = \alpha[\alpha + (1-\alpha)k^{-\gamma}]^{\frac{1-\gamma}{\gamma}}$. With $\gamma \in (0,1]$, $\lim_{k\to 0} f'(k) = \infty$ obviously holds. However, the second requirement of the Inada conditions, $\lim_{k\to\infty} f'(k) = 0$, does not hold in this case. Precisely, we have $\lim_{k\to\infty} f'(k) = \alpha^{1/\gamma}$.
- Fortunately, with $\alpha^{1/\gamma} < \frac{n+\delta+g}{s}$, we make sure $\lim_{k\to\infty} sf'(k) < n+\delta+g$. And this property, together with all the results above, guarantees that there is a unique steady state for the system.
- The proof and illustration of the unique steady state is thus the same as the process of equations (1.10)-(1.18) and Figure 1.2 or 1.3 in your textbook! You only need to substitute the specific form of function $f(\cdots)$ into them!
- You , however, need to make sure that the $sf(k)$ curve does not cross point $(0,0)$ in your graph.

1.(c)

$$sf(k^*) = (n+\delta+g)k^* \Longrightarrow k^* = \left[\frac{(\frac{n+\delta+g}{s})^\gamma - \alpha}{1-\alpha}\right]^{-\frac{1}{\gamma}}$$

1.(d) Yes. Balanced growth property same as in your textbook.

这 50 分是送分题。。。

2.(a)（2.44）now becomes:

$$\begin{aligned} c_{2t+1} &= (1+r_{t+1})(w_t A_t - C_{1t} - d) + (1+\xi_{t+1})d \\ &= (1+r_{t+1})(w_t A_t - C_{1t}) + (\xi_{t+1} - r_{t+1})d \end{aligned} \tag{2}$$

2.(b) No. It is straightforward to check from (2) that the security system does not make any difference to the budget constraint if $\xi_t \equiv r_t$. As a result, it cannot change the economic results.

2.(c) Hint: Population grows at rate n.

2.(d) Hint: Following the same logic embedded in (2.50)-(2.60) yields

$$C_{1t} = \frac{1+\rho}{2+\rho} w_t A_t + \frac{1+\rho}{(1+r)(2+\rho)}(\xi - r)d, \tag{3}$$

which gives us

$$\begin{aligned} K_{t+1} &= L_t A_t w_t - C_{1t} L_t - dL_t \\ &= L_t \left[\frac{w_t A_t}{2+\rho} - \frac{1+\rho}{(1+r)(2+\rho)}(\xi - r)d - d\right] \end{aligned} \tag{4}$$

Finally, we get

$$k_{t+1} = \frac{1-\alpha}{(1+n)(1+g)(2+\rho)} k_t^{\alpha} - \frac{d}{(1+n)A_{t+1}} \left(\frac{1+\rho}{2+\rho} \cdot \frac{\xi - r}{1+r} + 1 \right) \tag{5}$$

2.(e) Hint: Note that compared to (2.60) in your textbook, the revised version, (5), includes an additional term:

$$-\frac{d}{(1+n)A_{t+1}} \left(\frac{1+\rho}{2+\rho} \cdot \frac{\xi - r}{1+r} + 1 \right).$$

And obviously, if $\xi > r$, we have that the revised curve $k_{t+1}(k_t)$ is below the original one in figure 2.11 of your textbook. This is straightforward to see if we have further $g = 0$ i.e., $A_t \equiv A$: We will have a lower k^* in the new steady state. For this reason, the dynamic inefficiency problem could be (partly) solved. Even for general situations $g > 0$, the introduction of the security system still can partly solve the dynamic inefficiency problem: Compared to the benchmark case, everyone has a looser budget constraint, i.e., is better off, with nobody being worse off.

As I mentioned in class, if the security system simply transfers wealth from younger generation to older ones in each state, i.e., $\xi = n$, the system can provides a Pareto improvement to solve the dynamic inefficiency problem only if the population grows fast enough, i.e., $n > r$.

Extension: A formal version of the model usually sets the magnitude of benefits as d_t, which is also growing at rate g. This simplified version ($d_t \equiv d$), however, provides some basic ideas about how the government can take advantage of a growing population to solve the dynamic inefficiency problem.