Advanced Macroeconomics
Midterm Exam II (Open-Book)
Undergraduate Program in Economics, HUST
Thursday, May/21/2015

Name: _____ **Student ID:**_____

1. (15+20=35 points) The following two questions are based on materials in Section 3.5 of your textbook:

(a) Recall that in order to obtain a Balanced-Growth-Path in which $\frac{\dot{C}(t)}{C(t)} = \frac{1-\phi}{\phi}BL_A$, we must have the necessary condition (3.40):

$$r(t) = \rho + \frac{\dot{C}(t)}{C(t)} = \rho + \frac{1-\phi}{\phi}BL_A.$$

For models in Chapters 1 and 2, interest rates are determined by the marginal product of capital. However, we do not have capital in this version of Romer's model. How does the interest rate occur? (Hint: Suppose an individual borrows ΔC units of wealth from others in the current period, why is she willing to pay more than ΔC back in the next period, even if $\rho = 0$?)

(b) Now add a government into the model, and suppose that it imposes a **profit-tax** at rate λ on all intermediate firms. In other words, if we still use $\pi(i,t)$ to denote the raw profits earned by the creator of idea i at time t, the firm receives only $(1-\lambda)\pi(i,t)$ after paying the profit taxes. The remaining settings of the model are the same as in your textbook. What is the revised version of equation (3.44)? How does λ affect L_A in the equilibrium?

2. (7+8=15 points) The following questions are based on materials in Chapters 3 and 4.

(a) Is the statement below True or False? The terminology "Endogenous Growth" simply means that individuals' consumption and saving decisions are made endogenously, depending on their aim to maximize their life-time utility under the No-Ponzi constraint.

(b) Are equations (4.13) and (4.15) in your textbook essentially the same? If no, explain why. If yes, what message is the author delivering through the comparison between them?

3. (5+10+10=25 points) The following three questions are based on Chapter 5.

(a) Explain equation (5.23). Rewrite it such that c_{t+2} and r_{t+2} both appear in the right-hand-side of it.

(b) Does the simplified model in Section 5.5 provide any unrealistic predictions, i.e., any results that are inconsistent to our observations in reality?

(c) Consider this statement: Technological progresses often raise wage rates, so people often choose to work more during periods with higher technological growth rates and take more leisure during other periods. Which figure in your textbook is consistent to the scenario depicted by the statement above?

4. (10+15=25 points) The following questions are based on materials in Chapter 6. Consider the economic system below:

$$\pi_t = \lambda y_t + \lambda \mu_t^{\pi}, \ \lambda > 0, \tag{1}$$

$$r_t = b\pi_t, \ b > 0, \tag{2}$$

$$y_t = E_t[y_{t+1}] - \frac{1}{\theta} r_t, \ \theta > 0, \tag{3}$$

$$\mu_t^{\pi} = \rho_{\pi} \mu_{t-1}^{\pi} + e_t^{\pi}, \ -1 < \rho_{\pi} < 1, \tag{4}$$

where e_t^{π} is the white noise.

(a) Where are conditions (1), (2) and (3) from? What are their formal names?

(b) Solve this problem. How does a monetary shock, μ_t^{π}, affects the equilibrium output and inflation?

Solutions and Hints

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I did not have enough time to double-check this file while writing it. So please do me a favor by pointing out the mistakes and typos, if any, that I have made. (just send an email to yiming@hust.edu.cn)

1.(a) In this over-simplified model, $C(t)$ grows at rate $\frac{1-\phi}{\phi}BL_A$, so the marginal utility of consumption, $C(t)$, declines at the same rate, which in turn means that individuals value the same amount of wealth less tomorrow than today! So the borrower is willing to pay back more, and the lender has to request more, in the next period. By the way, the term interest rate is indeed used to measure the rate of return for any wealth that is transferred intertemporally, in the models with capital, a proportion of the final products is explicitly transferred to the next period in the form of capital, that is why we relate the interest rate to the return rate of capital in other models.

Let's rephrase the paragraph above in this way: the Euler equation that relates consumption growth to interest rate gives the supply function of capital, and the neo-classical distribution rule which relates MPK to interest rate defines the demand function of capital. In a model without capital, the demand of capital is always 0 for any r. And an appropriate r should also generates exactly 0 supply of capital. That is how we get equation (3.40).

1.(b) As long as you understand the specific equilibrium path we are interested in and how we obtain (3.43) from (3.37), you can have the correct answer straightforward.

Now (3.37) becomes:

$$\Pi(t) = (1-\lambda)\int_{\tau=t}^{\infty} e^{-r(\tau-t)}\pi(i,\tau)\,d\tau = \frac{w(t)}{BA(t)}. \tag{5}$$

Equations (3.38) − (3.40) remain the same after introducing the profit-tax policy (why?). So (3.41) becomes:

$$\Pi(t) = \frac{(1-\phi)(1-\lambda)}{\phi}\frac{\bar{L}-L_A}{\rho+BL_A}\frac{w(t)}{A(t)}. \tag{6}$$

Substituting (6) into (5) gives us the revised version of (3.44):

$$L_A = \max\left\{\frac{(1-\phi)(1-\lambda)\bar{L}B-\phi\rho}{(1-\phi)(1-\lambda)B+B\phi},\,0\right\}. \tag{7}$$

It is thus easy to check $\frac{\partial L_A}{\partial \lambda} < 0$. You can achieve this either by writing out the formal definition of this partial derivative, or simply by dividing the denominator and the numerator by $(1-\lambda)$ and check L_A's monotonicity in λ.

2.(a) False. "Endogeneity" is for technological growth, not for consumption decisions.

2.(b) Yes they are essentially the same. In applied studies, researchers often use (4.15 rather than (4.13). The reason is explained in your textbook.

3.(a) Euler equation with uncertainties. Consider an arrangement in which a household consumes less in period t and consumes more in period $t+2$ rather than in $t+t$ as in the textbook. Then the new Euler equation becomes:

$$\frac{1}{c_t} = e^{-2\rho} E_t \left[\frac{1}{c_{t+2}} (1 + r_{t+1})(1 + r_{t+2}) \right]. \tag{8}$$

3.(b) Yes it does, e.g., in this model, consumption and investment always fluctuate at the same rate. This is inconsistent to the observations in Table 5.2 of your textbook.

3.(c) Figure 5.2, the *wealth* and *intertemporal-substitution* effects, respectively.

4.(a) Phillips curve or revised Phillips curve, or Expectations-augmented Phillips curve for (1). MP curve, or LM curve, or Taylor's rule for (2). Is curve for (3).

4.(b) Substituting (1) and (2) into (3) yields

$$y_t = \frac{\theta}{\theta + b\lambda} E_t \left[y_{t+1} \right] - \frac{b\lambda}{\theta + b\lambda} \mu_t^{\pi}. \tag{9}$$

Define $x = \frac{\theta}{\theta + b\lambda}$, so $1 - x = \frac{b\lambda}{\theta + b\lambda}$. Adopting the **law of iterated projections** and

the AR-1 process (4) to (9) gives us

$$\begin{aligned}
y_t &= xE_t\left[y_{t+1}\right] - (1-x)\mu_t^{\pi} \\
&= xE_t\left[E_{t+1}\left(y_{t+2}\right) - (1-x)\mu_{t+1}^{\pi}\right] - (1-x)\mu_t^{\pi} \\
&= xE_t\left[y_{t+2}\right] - x(1-x)E_t\left[\mu_{t+1}^{\pi}\right] - (1-x)\mu_t^{\pi} \\
&= xE_t\left[y_{t+2}\right] - (1-x)(1+\rho_{\pi}x)\mu_t^{\pi} \\
&= \cdots \quad \cdots \\
&= \lim_{\tau\to\infty}\left\{x^{\tau}E_t\left[y_{t+\tau}\right] - (1-x)(1+\rho_{\pi}x+\rho_{\pi}^2x^2+\rho_{\pi}^3x^3+\cdots+\rho_{\pi}^{\tau}x^{\tau})\mu_t^{\pi}\right\} \\
&= \lim_{\tau\to\infty} x^{\tau}E_t\left[y_{t+\tau}\right] - (1-x)\mu_t^{\pi}\sum_{\tau=0}^{\infty}(\rho_{\pi}x)^{\tau} \\
&= -\frac{1-x}{1-\rho_{\pi}x}\mu_t^{\pi}\,, \ for\ \rho_{\pi}x \in (-1,1). \quad (fundamental\ solution) \qquad (10)
\end{aligned}$$

Substituting (10) into (1) yields

$$\begin{aligned}
\pi_t &= \lambda y_t + \lambda\mu_t^{\pi} \\
&= \frac{(1-\rho_{\pi})x}{1-\rho_{\pi}x}\lambda\mu_t^{\pi} \qquad (11)
\end{aligned}$$

Obviously, no matter the sign of ρ_{π}, a positive shock always decreases the output and increases the inflation.

Discussion: Term μ_t^{π} could be interpreted either as a opposite monetary shock (why?), or as a supply shock (ε_t^S) in (6.22) of your textbook. Let us take the latter way first, a supply shock can lead to a lower output level and a higher inflation rate at the same time (recalling the stagflation in the 1970s of U.S. economy, as explained in contexts around Figure 6.7?) Now consider the first direction, under what conditions does a positive monetary shock improve output as well as lead to a higher inflation rate? Obviously, if we have $b < 0$ (thus $x > 1$) as well as $|\rho_{\pi}x| < 1$, we can observe the "ordinary" result of a higher output and a higher inflation induced by a positive shock. Meaning? Inflation itself cannot lead to a higher inflation if the central bank tightly follows a prudent interest-rate rule (anchoring at the inflations). Equivalently speaking, we need some specific nominal price rigidity to make inflation attractive: If the nominal capital price (nominal interest rate i_t) is rigid, then we actually have $b < 0$!