Advanced Macroeconomics

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Midterm Exam II (Open-Book)

Undergraduate Program in Economics, HUST

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Name: ______________________ **Student ID:**______________________

1. ($20' \times 5 = 100$ points) All the following questions are based on Section 3.5 of your textbook.

(a) Consider Equation (3.30) in your textbook:

$$\mathcal{L} = \int_{i=0}^{A} p(i)L(i)\,\mathrm{d}i - \lambda\left\{\left[\int_{i=0}^{A} L(i)^{\phi}\,\mathrm{d}i\right]^{1/\phi} - 1\right\}. \tag{3.30}$$

It is said that the *Lagrangian* above depicts the problem of producing **one unit** of output at minimum cost. But a representative output producer could naturally produce more than one unit of output. So why should we focus on solving the problem of (3.30)?

(b) Recall that Equation (3.32) is a necessary condition for solving the Lagrangian of (3.30). It is said that λ in (3.32) could be regarded as an overall price level. Find the explicit expression for it. (Hint: ♠ Use the fact that the representative producer is producing one unit of output. ♣ Your answer should be in the form of $\lambda = [\int_{i=0}^{A} f(p(i))\,\mathrm{d}i]^{g(\phi)}$, where $f(\cdot)$ and $g(\cdot)$ are smooth functions for $\phi \in (0,1)$.)

(c) Denote by $\bar{p}$ the λ you found above, then Equation (3.32) could be rewritten as:

$$L(i) = \left[\frac{p(i)}{\bar{p}}\right]^{\frac{1}{\phi-1}}, \quad \text{where} \quad \phi \in (0,1). \tag{3.32'}$$

it simply means that the i-th input's demand is a decreasing function of its *relative price*. Now suppose you are the i-th idea's owner, and you know that the demand curve for your product is depicted as in (3.32′) above, your marginal cost is fixed at c. As a monopolist, how would you set the price for your product? How will your decision change the overall price level?

(d) There is a typo (印刷错误) in Equation (3.41) of your textbook. Point it out.

(e) Suppose that, instead of imposing a logarithmic instantaneous utility function, individuals are having a generic $CRRA$ one. That is to say, Equation (3.35) of your textbook becomes:

$$U = \int_{t=0}^{\infty} e^{-\rho t} \frac{C(t)^{1-\theta}}{1-\theta}\,\mathrm{d}t, \quad \rho > 0, \quad \theta > 0. \tag{3.35'}$$

Given this change, what is the revised version of Equation (3.44)? How does θ affect equilibrium L_A? Explain your answer with economic intuitions.

(a) 由于厂商的生产函数是规模报酬不变的，且成本函数是关于要素投入的线性函数，故可将生产 m 个单位产出的成本最小化问题标准化为生产1个单位产出的成本最小化问题。

$$\mathcal{L}' = \int_{i=0}^{A} p(i)L(i)di - \mu\{[\int_{i=0}^{A} L(i)^{\phi}di]^{\frac{1}{\phi}} - m\}$$

$$\frac{\partial \mathcal{L}'}{\partial L(i)} = p(i) - \mu\frac{1}{\phi}[\int_{i=0}^{A} L(i)^{\phi}di]^{\frac{1}{\phi}-1}\cdot\phi L(i)^{\phi-1} = 0$$

将 $[\int_{i=0}^{A} L(i)^{\phi}di]^{\frac{1}{\phi}} = m$ 代入，可得

$$p(i) = \mu(m)^{1-\phi}L(i)^{\phi-1}$$

$$L(i) = \left(\frac{\mu m^{1-\phi}}{p(i)}\right)^{\frac{1}{1-\phi}}$$

$$= \left(\frac{\mu}{p(i)}\right)^{\frac{1}{1-\phi}} m$$ 与生产1个单位产品的成本最小化问题的一阶条件 $\underbrace{\text{是线性关系且}}_{\text{所导出的需求函数}}$ 需求价格弹性不变。

(b) $[\int_{i=0}^{A} L(i)^{\phi}di]^{\frac{1}{\phi}} = 1$

将 $L(i) = \left(\frac{\lambda}{p(i)}\right)^{\frac{1}{1-\phi}}$ 代入，可得 $[\int_{i=0}^{A}(\frac{\lambda}{p(i)})^{\frac{\phi}{1-\phi}}di]^{\frac{1}{\phi}} = 1$

$$\lambda^{\frac{1}{1-\phi}}[\int_{i=0}^{A} p(i)^{-\frac{\phi}{1-\phi}}di]^{\frac{1}{\phi}} = 1$$

$$\lambda = [\int_{i=0}^{A} p(i)^{-\frac{\phi}{1-\phi}}di]^{-\frac{1-\phi}{\phi}}$$

(c) 首先证明垄断定价原则 $p = \frac{|\varepsilon|}{|\varepsilon|-1}MC$　　ε：需求价格弹性

$TR = p(q)\cdot q$

$$MR = p(q) + q\cdot\frac{dp(q)}{dq} = p(1+\frac{q}{p}\frac{dp}{dq}) = p(1+\frac{1}{\varepsilon}) = p(1-\frac{1}{|\varepsilon|}) = p\cdot\frac{|\varepsilon|-1}{|\varepsilon|}$$

由 $MR = MC$

可得 $p = \frac{|\varepsilon|}{|\varepsilon|-1}MC$

本题中，$|\varepsilon| = \frac{1}{1-\phi}$，$MC = c$，则 $p(i) = \frac{c}{\phi}$

$$\bar{p} = \lambda = [\int_{i=0}^{A}(\frac{c}{\phi})^{\frac{-\phi}{1-\phi}}di]^{-\frac{1-\phi}{\phi}} = A^{\frac{\phi-1}{\phi}}\frac{c}{\phi}$$

个人的定价决定不会影响总体的价格水平

①从积分角度看，i 对 $\bar{p}$ 的影响很小

②从经济直觉看，有无穷多个厂商，每个厂商价格设定对总体价格的影响可忽略不计

(d) $\pi(t)$ 应为 $\Pi(t)$，指的是利润的贴现值，形式为(3.37)左边

(e) 由于 CRRA 的效用形式

$$g_c = \frac{\dot{C}(t)}{C(t)} = \frac{r(t)-\rho}{\theta} \Rightarrow r(t) = \rho + \theta\frac{(1-\phi)}{\phi} B L_A$$

$$\Pi(t) = \frac{\frac{1-\phi}{\phi}(\bar{L}-L_A)\frac{W(t)}{A(t)}}{\rho + \frac{\theta(1-\phi)}{\phi} B L_A - \frac{1-2\phi}{\phi} B L_A}$$

$$= \frac{(1-\phi)(\bar{L}-L_A)}{\rho\phi + [\theta(1-\phi)-(1-2\phi)] B L_A} \cdot \frac{W(t)}{A(t)}$$

$$= \frac{(1-\phi)(\bar{L}-L_A)}{\rho\phi + [\theta-1+(2-\theta)\phi] B L_A} \cdot \frac{W(t)}{A(t)}$$

由于 $\Pi(t) = \frac{W(t)}{BA(t)}$

可得 $\frac{(1-\phi)(\bar{L}-L_A)}{\rho\phi + [\theta-1+(2-\theta)\phi] B L_A} = \frac{1}{B}$

$$L_A = \frac{(1-\phi)B\bar{L} - \rho\phi}{(\theta+\phi-\theta\phi)B}$$

$$= \frac{(1-\phi)B\bar{L} - \rho\phi}{B[\phi+\theta(1-\phi)]}$$

$$L_A = \max\left\{\frac{(1-\phi)B\bar{L} - \rho\phi}{B[\phi+\theta(1-\phi)]}, 0\right\}$$

相对风险规避系数 θ 越大，L_A 越小，从事研发的人数越少。

相对风险规避系数 θ，在没有不确定性的模型中，不再理解为对风险的厌恶程度，而理解为对跨期消费的厌恶程度。θ 较大时，个体对更高的消费波动（消费增长率）更为厌恶，因此均衡下的 L_A 越小（因为 L_A 的值事实上决定了 A, Y, C 的增长率）。